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FOLIATIONS AND THE TOPOLOGY OF 3-MANIFOLDS[1]

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In this announcement we discuss the close relationship between the topology of 3-manifolds and the foliations that is possesses. We will introduce and state the main result, then use it and the ideas of its proof to state some geometric and topological corollaries. Details to almost all the results can be found in [$\mathbf{G_4}$].

Given a compact, connected, oriented 3-manifold, when does there exist a codimension-1 transversely oriented foliation $\mathcal{F}$ which is transverse to ∂M and has no Reeb components? If such an $\mathcal{F}$ exists ∂M necessarily is a (possibly empty) union of tori and M is either $S^2 \times S^1$ (and $\mathcal{F}$ is the product foliation) or irreducible. The first condition follows by Euler characteristic reasons while the latter basically follows from the work of Reeb [**Re**] and Novikov [**N**] although first observed by Rosenberg [**Ro**]. Our main result says that such conditions are sufficient when $H_2(M, \partial M) \neq 0$.

If such a foliation $\mathcal{F}$ exists on M then it follows from the work of Thurston [$\mathbf{T_1}$] that any compact leaf L is a Thurston norm minimizing surface [i.e., $|\chi(L')| \leq |\chi(T')|$ for any properly embedded T with $[T] = [L] \in H_2(M, \partial M)$ (or $H_2(M)$ if we were discussing the norm on $H_2(M)$), where S' denotes S-(sphere and disc components)] for the class $[L] \in H_2(M, \partial M)$. Our main result says that for a 3-manifold M satisfying the above necessary conditions any norm minimizing surface can be realized as a compact leaf of a foliation without Reeb components.

THEOREM. *Let M be a compact, connected, irreducible, oriented 3-manifold whose boundary is a (possibly empty) union of tori i.e., M is Haken and $\chi(M) = 0$. Let S be any norm minimizing surface representing a nontrivial class $z \in H_2(M, \partial M)$. Then there exists foliations $\mathcal{F}_0$ and $\mathcal{F}_1$ of M such that*

(1) *for $i = 0, 1$, $\mathcal{F}_i \pitchfork \partial M$ and $\mathcal{F}_i | \partial M$ has no Reeb components,*
(2) *every leaf of $\mathcal{F}_0$ and $\mathcal{F}_1$ nontrivially intersects a closed transverse curve,*
(3) *S is a compact leaf of both $\mathcal{F}_0$ and $\mathcal{F}_1$,*
(4) *$\mathcal{F}_0$ is of finite depth,*
(5) *$\mathcal{F}_1$ is C^∞ except possibly along toral components of S.*

COROLLARY. *Let L be an oriented nonsplit (i.e., no embedded S^2 in $S^3 - L$ separates the components of L) link in S^3. Then S is a surface of minimal genus for L if and only if there exists a C^∞ transversely oriented foliation $\mathcal{F}$ of $S^3 - \mathring{N}(L)$ such that*

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