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ON THE VANISHING OF POINCARÉ SERIES OF RATIONAL FUNCTIONS

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1. Let Γ be a finitely generated nonelementary Kleinian group with region of discontinuity Ω and limit set Λ. Let $\lambda(z)|dz|$ be the Poincaré metric on Ω (normalized to have constant negative curvature -1). Let $q \in \mathbf{Z}$, $q \geq 2$. A *cusp form for* Γ *of weight* $(-2q)$ is a holomorphic function φ on Ω satisfying

$$\varphi(\gamma z)\gamma'(z)^q = \varphi(z), \quad \text{for all } \gamma \in \Gamma, \text{ for all } z \in \Omega, \tag{1}$$

and either (hence both) of the following equivalent conditions:

$$\iint_{\Omega/\Gamma} \lambda(z)^{2-q}|\varphi(z)\, dz \wedge d\bar{z}| < \infty; \tag{2}$$

$$\sup_{z\in\Omega}\{\lambda(z)^{-q}|\varphi(z)|\} < \infty. \tag{3}$$

The equivalence of (2) and (3) shows that the *Peterson scalar product*

$$\langle \varphi, \psi \rangle = i \iint_{\Omega/\Gamma} \lambda(z)^{2-2q}\varphi(z)\overline{\psi(z)}\, dz \wedge \overline{dz} \tag{4}$$

induces a Hilbert space structure on the space of cusp forms.

Let Δ be a Γ-invariant union of components of Ω, and define $\mathbf{A}_q(\Delta)$ to be the space of cusp forms for Γ of weight $(-2q)$ that vanish on $\Omega\backslash\Delta$. Abbreviate $\mathbf{A}_q(\Omega)$ by $\mathbf{A}_q$.[2]

Define R_q to be the space of rational functions f such that
(5) f is holomorphic on Ω,
(6) f has only simple poles (on Λ), and

$$\begin{aligned} f(z) &= O(|z|^{-2q}), \quad && z \to \infty \text{ if } \infty \in \Omega, \text{ and} \\ f(z) &= O(|z|^{-(2q-1)}), \quad && z \to \infty \text{ if } \infty \in \Lambda. \end{aligned} \tag{7}$$

If $f \in R_q$, then the *Poincaré series*

$$\sum_{\gamma\in\Gamma} f(\gamma z)\gamma'(z)^q, \qquad z \in \Omega, \tag{8}$$

converges absolutely and uniformly on compact subsets of Ω and defines a cusp form $\Theta_q f \in \mathbf{A}_q$. Bers [**3**] has shown that

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[2]The group Γ is fixed throughout this paper. We hence suppress in the notation the dependence on Γ of the various spaces and operators considered.

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