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# ORTHOGONAL POLYNOMIALS ASSOCIATED WITH INVARIANT MEASURES ON JULIA SETS

BY M. F. BARNSLEY[1], J. S. GERONIMO[2] AND A. N. HARRINGTON

**I. Introduction.** Let **C** be the complex plane and $T$: $\mathbf{C} \longrightarrow \mathbf{C}$ be a nonlinear polynomial of the form $T(z) = z^n + k_1 z^{n-1} + \cdots + k_n$. Consider the sequence $\{T^n(z)\}$, $n = 1, 2, \ldots$, where $T^0(z) = z$ and $T^n(z) = T \circ T^{n-1}(z)$. The general theory of sequences of this form has been developed by Fatou [5, 6], Julia [8] and Brolin [4]. In their research a fundamental role is played by the Julia set $B$, which is the set of points in **C** where $\{T^n(z)\}$ is not normal.

Fatou and Julia established the possible structures $B$ can have and showed that these depend in a complicated way on the coefficients of $T(z)$. Among other things they demonstrated that $B$ may be the unit circle, a straight line, a generalized Cantor set, or a set containing an infinite number of Jordan curves. However, in all cases, $B$ is compact and $T^{-1}(B) = B$.

In 1965 Brolin established some electrical properties of the set $B$. He showed that the logarithmic capacity of $B$ is positive and that there exists an equilibrium charge distribution $u$. He also proved that, for $T$: $B \longrightarrow B$, $u$ is invariant and the system $(B, u, T)$ is strongly mixing. It is the purpose of this note to develop more fully the properties of the equilibrium measure on $B$ and to investigate the monic polynomials orthogonal with respect to this measure.

**II. Results.** We begin with the following

DEFINITION 1. $u$ is a balanced $T$-invariant measure on $B$ if $u$ is a probability measure supported on $B$ such that for any complete assignment of branches of $T^{-1}$, $T_j^{-1}$, $j = 1, 2, \ldots, n$, $u(T_j^{-1}(S)) = u(S)/n$ for each Borel set $S$.

REMARK 1. One can show [2] that there is only one balanced $T$-invariant measure on $B$ and that the measure constructed by Brolin is balanced.

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