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NORMAL, NOT PARACOMPACT SPACES

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ABSTRACT. We describe some recently constructed counterexamples in general topology, including a normal, nonmetrizable Moore space, a normal para-Lindelöf, not paracompact space, and a normal, screenable, not paracompact space.

The period 1948–1952, when the notions of paracompactness and metrizability were investigated in terms of discrete and locally finite collections, was a period of great progress in point set topology. The work includes beautiful theorems, important counterexamples, and natural, unanswered questions. For example, Michael [M] showed that a space is paracompact (i.e. every open cover has a locally finite open refinement) if every open cover has a σ-locally finite open refinement. Can "locally countable" or "σ-disjoint" replace "σ-locally finite"? What if the space is normal? Bing's example B (see [B]) is a screenable, metacompact Moore space which is not paracompact. Is there such an example which is normal? The purpose of this announcement is to describe the series of papers [$\mathbf{F_1}$, N, $\mathbf{F_2}$, $\mathbf{F_3}$, R] which answer the above and similar questions by constructing normal, not paracompact spaces.

Let us review some definitions. In this paper we consider only regular, T_3 spaces. A collection of subsets of a space X is locally finite (resp. locally countable) if every $x \in X$ has a neighborhood meeting finitely many (resp. countably many) elements of the collection. A collection has the σ-property if it is the union of countably many collections with the property. A space is screenable (resp. para-Lindelöf) if every open cover has a σ-disjoint (resp. locally countable) open refinement. A More space is a special type of first countable space; we will not need the precise definition.

We begin by describing the nonseparable metric space, F, from which the spaces are constructed. Points of F are functions from ω (the set of natural numbers) to ω_1 (the set of countable ordinals). The distance, $d(f, g)$ between two points of F is 2^{-n}, where n is least such that $f(n) \neq g(n)$.

Let Σ_n be the set of functions to ω_1 with domain $\{0, 1, 2, \ldots, n-1\}$. For $\sigma \in \Sigma_n$ we define $N_\sigma = \{f \in F\colon \sigma \subset f\} = \{f \in F\colon f(0) = \sigma(0), \ldots, f(n-1) = \sigma(n-1)\}$. Then $\{N_\sigma\colon \text{for some } n \in \omega, \sigma \in \Sigma_n\}$ is a base for F. We

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