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# NON-EUCLIDEAN FUNCTIONAL ANALYSIS AND ELECTRONICS

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**1. Introduction.** We shall describe a structure built from several components —functional analysis, a few symmetric spaces, a Lie group over a function field, and Nevanlinna-Pick interpolation theory—all fixed on a framework of engineering motivation which determines the relationship between them. A large branch of functional analysis concerns linear spaces of analytic functions called $H^p$ spaces and linear operators on them. Here we describe an analogous study for some sets of functions which while not linear have a very rigid structure patterned on that of the Poincaré disk. There are several basic results in linear $H^p$ theory which have good "non-Euclidean" analogs. One is the classical Szegö theorem which computes the $L_2(d\mu)$ distance of a function $f$ to $H^2$. Actually our non-Euclidean result is closer to a theorem of Nehari which computes the supremum norm distance of an $f$ in $L^\infty$ to $H^\infty$. Another is the Beurling-Lax-Halmos theorem from which such things as the existence of Weiner-Hopf factorizations and the F. and M. Riesz theorem immediately follow.

The rigidity of a geometry on a space is expressed in terms of the group of isometries on that space. These are thought of as rigid structure preserving motions and an early notion of geometry championed by Felix Klein was to specify a space, a group of motions on it, and then to study invariants of these motions. That viewpoint seems highly appropriate for the physical situations we shall encounter.

Recall that the Poincaré disk is the unit ball

$$\mathcal{B}\mathbf{C} = \{z : |z| < 1\}$$

of $\mathbf{C}$ along with the linear fractional transformations

$$\mathcal{G}_g(s) = (\alpha s + \beta)(\kappa s + \gamma)^{-1}$$

of $\mathcal{B}\mathbf{C}$ onto itself. Each $\mathcal{G}_g$ in this group of rigid motions has a coefficient matrix $g$ which satisfies

$$g^*\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} g = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; \tag{1.0}$$

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