

BOOK REVIEWS

Cartanian geometry, nonlinear waves, and control theory, by Robert Hermann, Interdisciplinary Mathematics, volumes 20 and 21, Math. Sci. Press, Brookline, Mass., Part A, 1979, xv + 501 pp., \$50.00; Part B, 1980, xii + 585 pp., \$60.00.

Cartanian geometry, A and B are the twentieth and twenty-first volumes in the series *Interdisciplinary mathematics* (IM) which is authored by Robert Hermann. (A companion series is *Lie groups: history, frontiers and applications*.)

The volumes under review represent both a refinement and an extension of earlier work in the IM series. Most particularly, for the purpose of this review, we need to refer to Vol. 3 (*Algebraic topics in system theory*), Vol. 8 (*Linear system theory and Introductory algebraic geometry*), Vol. 9 (*Geometric structure of systems—Control theory and physics, Part A*), Vol. 11 (*Geometric structure of systems—Control Theory, Part B*) and Vol. 13 (*Algebro—Geometric and Lie theoretic techniques in systems theory, Part A—coauthored with Clyde Martin*). As is the case with most of the series, *Cartanian Geometry A,B* cover an enormous number of topics. Broadly grouped, they fall into the categories (1) systems and control theory; (2) nonlinear waves; (3) quantum mechanics and (4) a translation (by Michael Ackerman) of Sophus Lie's papers *General investigations of differential equations which admit a finite continuous group* and *Foundations of the theory of infinite continuous transformation groups*. I. This last item is of value to historians of modern mathematics as well as those whose interest is in Lie groups or differential geometry.

Because of the wealth of ideas dealt with in these volumes and the need to keep this review to a manageable length, attention will be restricted to the material on the matrix Riccati equation and on the relationship between control/system theory and vector bundle theory. (Those interested in the topic of nonlinear waves may wish to see Hermann's review [33] of *Elements of soliton theory* by G. L. Lamb, Jr. The presentation in Lamb's book is from a different perspective from that of Hermann's work in this area, but Hermann has taken the opportunity of the review to give an overview of the geometric theory of nonlinear waves.)

Before we discuss in depth any of the mathematical content, a few words are in order about the author's philosophy. Perhaps the first two paragraphs of the preface to Part A best sum up his attitude.

"I began in 1970 to write this series of books in order to develop a *unified* mathematical science and technology. After all, if subjects like category theory, logic, differential topology are accepted and integrated into the mathematical world, why not system theory, mathematical elementary particle theory,