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THREE DIMENSIONAL MANIFOLDS, KLEINIAN GROUPS AND HYPERBOLIC GEOMETRY

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1. A conjectural picture of 3-manifolds. A major thrust of mathematics in the late 19th century, in which Poincaré had a large role, was the uniformization theory for Riemann surfaces: that every conformal structure on a closed oriented surface is represented by a Riemannian metric of constant curvature. For the typical case of negative Euler characteristic (genus greater than 1) such a metric gives a hyperbolic structure: any small neighborhood in the surface is isometric to a neighborhood in the hyperbolic plane, and the surface itself is the quotient of the hyperbolic plane by a discrete group of motions. The exceptional cases, the sphere and the torus, have spherical and Euclidean structures.

Three-manifolds are greatly more complicated than surfaces, and I think it is fair to say that until recently there was little reason to expect any analogous theory for manifolds of dimension 3 (or more)—except perhaps for the fact that so many 3-manifolds are beautiful. The situation has changed, so that I feel fairly confident in proposing the

1.1. CONJECTURE. *The interior of every compact* 3-*manifold has a canonical decomposition into pieces which have geometric structures.*

In §2, I will describe some theorems which support the conjecture, but first some explanation of its meaning is in order.

For the purpose of conservation of words, we will henceforth discuss only oriented 3-manifolds. The general case is quite similar to the orientable case.

1. The decomposition referred to really has two stages. The first stage is the prime decomposition, obtained by repeatedly cutting a 3-manifold M^3 along 2-spheres embedded in M^3 so that they separate the manifold into two parts neither of which is a 3-ball, and then gluing 3-balls to the resulting boundary components, thus obtaining closed 3-manifolds which are "simpler". Kneser [**Kn**] proved that this process terminates after a finite number of steps. The resulting pieces, called the prime summands of M^3, are uniquely determined by M^3 up to homeomorphism; cf. Milnor, [**Mil 1**].

The second stage of the decomposition involves cutting along tori. This was discovered much more recently, by Johannson [**Joh**] and Jaco and Shalen [**Ja, Sh**], even though the elementary theory of the torus decomposition does not

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