

UNITS AND CLASS GROUPS IN NUMBER THEORY AND ALGEBRAIC GEOMETRY

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§1. Introductory remarks

Determining unramified coverings over various base spaces is a classical activity, which can take place in many contexts: topological, complex analytic, algebraic, and arithmetic. The abelian coverings are simpler to handle than the non-abelian ones, and in these lectures, we shall concentrate on abelian cases. Furthermore, the base space will have mostly dimension 1.

It turns out that the study of the arithmetic case is inextricably intertwined with that of the other cases, in many ways. Thus even though I (for example) start being motivated by the arithmetic case, I come eventually to a consideration of the other cases.

Presented as the Colloquium Lectures at the eighty-fifth summer meeting of the American Mathematical Society, in Pittsburgh, August 17–20, 1981; received by the editors July 8, 1981.

1980 *Mathematics Subject Classification*. Primary 12A35, 12A90; Secondary 14G25, 10D12.

¹Supported by NSF Grant.