BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 6, Number 2, March 1982

POINCARÉ AND LIE GROUPS

BY WILFRIED SCHMID[1]

When I was invited to address this colloquium, the organizers suggested that I talk on the Lie-theoretic aspects of Poincaré's work. I knew of the Poincaré-Birkhoff-Witt theorem, of course, but otherwise was unaware of any contributions that Poincaré might have made to the general theory of Lie groups, as opposed to the theory of discrete subgroups. It thus came as a surprise to me to find that he had written three long papers on the subject, in addition to several short notes. He evidently regarded it as one of the major mathematical developments of his time—the introductions to his papers contain some flowery praise for Lie—but one probably would not do Poincaré an injustice by saying that in this one area, at least, he was not one of the main innovators. Still, his papers are intriguing for the glimpse they give of the early stages of Lie theory. Perhaps this makes a conference on the work of Poincaré an appropriate occasion for some reflections on the origins of the theory of Lie groups.

Sophus Lie (1842–1899) developed his theory of *finite continuous transformation groups*, as he called them, in the years 1874–1893, in a series of papers and three monographs. To Lie, a transformation group is a family of mappings

$$y = f(x, a), \tag{1a}$$

where x, the independent variable, ranges over a region in a real or complex Euclidean space; for each fixed a, the identity (1a) describes an invertible map; the collection of parameters a also varies over a region in some $\mathbf{R}^n$ or $\mathbf{C}^n$; and f, as function of both x and a, is real or complex analytic. Most importantly, the family is closed under composition: for two values a, b of the parameter, the composition of the corresponding maps belongs again to the family, i.e.,

$$f(f(x, a), b) = f(x, c), \tag{1b}$$

with

$$c = \varphi(a, b) \tag{1c}$$

depending analytically on a and b, but not on x. It must be noted that these identities are only required to hold locally; in present-day terminology, (1a–c) define the germ of an analytic group action.

Presented to the Symposium on the Mathematical Heritage of Henri Poincaré, April 7–10, 1980; received by the editors November 1, 1980.

1980 *Mathematics Subject Classification*. Primary

[1]Supported in part by NSF grant MCS-79-13190.