BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 5, Number 1, July 1981

RESEARCH ANNOUNCEMENTS

ON REPRESENTATIONS IN COHOMOLOGY OVER PSEUDO-HERMITIAN SYMMETRIC SPACES

BY C. M. PATTON

A significant part of the problem of classifying representations of groups is discovering which representations are *unitarizable*, i.e. in which cases do the vector spaces of the representation admit an invariant Hilbert space structure. In large measure, the difficulty here is finding an appropriate realization of the abstract space of the representation as a concrete space of functions, forms, cohomology classes, etc. One class of representations where the unitary structure (if it exists) is not well understood is given by considering the natural action of groups on holomorphic, homogeneous line bundles over pseudo-hermitian symmetric spaces, and then the induced action on cohomology with coefficients in the sheaf of holomorphic sections of these line bundles.

It has been known for some time [1], [2] that the representations of $SU(2, 2)$ on certain cohomology spaces over $SU(2, 2)/S(U(1) \times U(1, 2))$ are equivalent to subrepresentations of the well-known "metaplectic" representation of $SU(2, 2)$, and are therefore unitarizable.

The purpose of this note is to present the analogous theorem for representations of $SU(p, q)$ in cohomology over $SU(p, q)/S(U(k) \times U(p - k, q))$.

Abstractly, we have the following situation: there is a pseudo-Kahler bundle, E, over the pseudo-hermitian symmetric space G/H, a Kahler bundle E' over the hermitian symmetric space G'/H' and an inclusion $i: E \hookrightarrow E'$ which preserves both the real affine connection and the real symplectic structure. This inclusion allows us to pull back holomorphic functions on E' to partially holomorphic functions on E, which we can then change to $\bar{\partial}$-closed forms and hence cohomology classes.

While G acts by Kahler isometries on E, and the action can be extended to all of E', G acts only by symplectic transformations on E' and not Kahler isometries. Hence, the geometric action of G on functions on E' will not take holomorphic functions to holomorphic functions. This can be remedied by using the metaplectic action of $G \subset$ (symplectic automorphisms of E'). An important fact is that the geometric action on cohomology, and the metaplectic transformations on functions then correspond.

Other cases in which such a generalized Harish-Chandra embedding exists

Received by the editors January 7, 1981.
1980 *Mathematics Subject Classification*. Primary 22E46, 32M15; Secondary 46E20.

0002-9904/81/0000-0304/$02.00