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DIFFERENTIABLE DYNAMICAL SYSTEMS AND THE PROBLEM OF TURBULENCE

BY DAVID RUELLE

1. Conservative and dissipative dynamical systems. The mathematical study of differentiable dynamical systems has its origin in the desire to understand the time evolutions which occur in nature. This relation to physical reality has exercised a strong influence on the progress of the subject. Other strong influences resulted from the internal development of mathematics and physics.

The time evolutions which one observes in nature yield two large classes of dynamical systems. On one hand we have frictionless mechanical systems, or *conservative systems*. On the other hand we have all kinds of natural systems where some "noble" form of energy is converted to heat; these are the *dissipative systems*.

Classical mechanics–the study of conservative systems–was initiated by Isaac Newton and has played a central role in the development of Natural Philosophy. In particular it led to the *Méthodes nouvelles de la mécanique céleste* of Henri Poincaré, and later to the solution of small denominator problems by Kolmogorov, Arnold and Moser. The importance of classical mechanics is due to the existence of nontrivial conservative systems–those of celestal mechanics–which can be investigated with extreme precision both observationally and theoretically.

The dissipative systems have a priori no less interest than the conservative ones. Mathematically they are more general; physically they have widespread occurrence, and display a great variety of phenomena. Unfortunately, dissipative systems do not obey, with great precision, laws as simple as those of celestial mechanics. They are usually continuous systems, requiring an infinite number of parameters for their description. Because of these and other difficulties, a detailed dynamical study of dissipative systems is taking place only now.

2. Viscous flows. Diffusion of heat, chemical reactions, heating of an electrical resistor, are examples of dissipative phenomena. So is the flow of a viscous fluid, where internal friction dissipates mechanical energy into heat.

Viscous flows are among the most remarkable natural phenomena. Whirlpools of the river, waves of the sea, dancing flames of the fire, twisting shapes of the clouds have enchanted poets and fascinated philosophers. To the scientist also viscous flows have something to offer, since understanding

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