BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 5, Number 1, July 1981

# SYMPLECTIC GEOMETRY

BY ALAN WEINSTEIN

**0. Introduction.** Classical mechanics in the time of Huygens (1629–1695) and Newton (1642–1727) was very geometrical. Although Newton invented the calculus in order to formulate and solve physical problems, many of his arguments made heavy use of euclidean geometry. After Newton, there came a period of "mécanique analytique," during which Lagrange (1736–1813) could boast that his treatise on mechanics contained no pictures.[1] Following the path of Euler (1707–1783) and Lagrange, Jacobi (1804–1851) and Hamilton (1805–1865) continued the development of analytic techniques for the explicit solution of the differential equations describing mechanical systems. Finally, geometry has taken a new role in mechanics through the contributions of Poincaré (1854–1912) and Birkhoff (1884–1944). Now, though, the geometry is the more flexible geometry of canonical (in particular, area preserving) transformations instead of the rigid geometry of Euclid; accordingly, the conclusions of the geometrical arguments are often qualitative rather than quantitative.

In this lecture (and paper), I would like to explain what symplectic geometry is and to describe its role in contemporary mathematics. I think it is not unfair to say that symplectic geometry is of interest today, not so much as a theory in itself, but rather because of a series of remarkable "transforms" which connect it with various areas of analysis.[2]

The *lagrangian submanifolds* play an especially important part in symplectic geometry and its applications. In §3 of this lecture, I will outline an approach to symplectic geometry in terms of a "category" in which the morphisms are exactly the lagrangian submanifolds; this approach suggests some interesting

Presented at the Symposium on the Mathematical Heritage of Henri Poincaré in April, 1980; received by the editors November 15, 1980.

1980 *Mathematics Subject Classification.* Primary 58F05.

[1] Poinsot (1777–1859) reacted strongly against this analytical tradition. Referring to "l'illustre Lagrange" in his famous study of rigid body rotation, Poinsot wrote that in Lagrange's treatment of the subject, "on ne voit guère que des calculs, sans aucune image nette de la rotation du corps." (I would like to thank J. J. Duistermaat for calling my attention to Poinsot's vivid critique of analytical mechanics, as well as for his comments on this manuscript.)

[2] G. D. Birkhoff's "disturbing secret fear that geometry may ultimately turn out to be no more than the glittering intuitional trappings of analysis" **[BI]** may be especially appropriate when applied to symplectic geometry. I learned of Birkhoff's statement from Chern [C], who tends to dismiss the fear on the grounds that Birkhoff was an analyst. The recent success of symplectic geometric methods in linear partial differential equations (see §5 for an example) suggests that one might need the glitter to find the gold. This opinion is also expressed by Poinsot (see previous footnote) who suggests that calculations are merely a tool in the service of geometrical and mechanical reasoning.


0002-9904/81/0000-0300/$04.25