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VARIATIONAL AND TOPOLOGICAL METHODS IN NONLINEAR PROBLEMS[1]

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These lectures are meant as an informal introduction to some of the techniques used in proving existence of solutions of nonlinear problems of the form

$$F(u) = y. \tag{1}$$

Here F is a continuous (and usually smooth) mapping from one topological space X to another Y, and the spaces are usually infinite dimensional. The model to keep in mind is one in which these spaces are function spaces defined in domains on finite-dimensional manifolds, and F is a system of nonlinear partial differential operators–or integral operators.

A number of special topics will be presented–in three parts:

I. Global methods: homotopy, in particular topological degree theory, and extensions. Applications to nonlinear boundary value problems.

II. Variational methods, in which a solution is a stationary point of some functional. Applications.

III. Local study, perturbation about a solution.

An important analytic aspect of all these problems is that of finding a priori estimates for the solutions. How one does that varies from problem to problem and I will barely touch on these technical aspects. I will try, rather, to avoid technicalities and stress the topological and variational ideas.

The lectures are not addressed to the experts in these fields–for them there will be little new. They are given with the hope of attracting others to the subject. Up to now, the topological and abstract ideas used are rather primitive, and I am confident that there will be enormous further development–involving more and more sophisticated topology.

A condensed version of some of this material was presented in **[48]**.

Here is a more specific list of the topics treated.

I.1 Some classical things. Continuity method. Degree theory.

I.2 Some recent extensions of Leray-Schauder degree theory.

I.3 Extension of degree theory to Fredholm maps by Elworthy and Tromba.

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