30. ______, *On affine-ruled rational surfaces*, Math. Ann. (to appear).

31. U. Storch, *Bemerking zu einem Satz von M. Kneser*, Arch. Math. (Basel) **23** (1972), 403–404.

32. A. A. Suslin, *On projective modules over polynomial rings*, Math. USSR Sb. **22** (1974).

33. W. van der Kulk, *On polynomial rings in two variables*, Nieuw Archief voor Wiskunde (3) **1** (1953), 33–41.

T. KAMBAYASHI

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 4, Number 2, March 1981

0002-9904/81/0000-0118/$02.75

Introduction to H_p spaces, by Paul Koosis, London Mathematical Society Lecture Note Series, No. 40, Cambridge University Press, Cambridge, New York, Melbourne, 1980, XV + 376 pp.

Suppose f is a function holomorphic on the open unit disk, Δ, of $\mathbf{C}$, and suppose $0 < p < \infty$. Then f is said to be in H_p if

$$\sup_{r<1}\left(\int_0^{2\pi}|f(re^{i\theta})|^p\frac{d\theta}{2\pi}\right)^{1/p} \equiv \|f\|_{H_p} < \infty.$$

H_∞ is the ring of bounded holomorphic functions on Δ and is endowed with the supremum norm. H_p spaces were first studied by Hardy (hence the terminology H_p) in 1915 and have remained an object of active study to this day. The past two decades have seen an enormous amount of work, and rather successful extensions of the H_p theory to $\mathbf{R}^n$, $\mathbf{C}^n$, and various other topological spaces have been made. The one dimensional theory remains interesting, however, for (at least) two reasons. Firstly, the existence of tools peculiar to one complex variable (e.g. conformal mappings and Blaschke products) makes life easier there than in higher dimensional spaces. Indeed, much of the current research in H_p spaces is devoted to finding analogues in $\mathbf{R}^n$ or $\mathbf{C}^n$ of theorems known in dimension one. Secondly, the space H_∞ has no known analogue in the $\mathbf{R}^n$ theory for $n \geqslant 2$, and the $\mathbf{C}^n$ theory seems to be extremely difficult when $n \geqslant 2$. An example of the latter phenomenon is the inner function problem: can there exist a nonconstant function f bounded and holomorphic on the unit ball of $\mathbf{C}^2$ such that f has radial limits of modulus one almost everywhere on the unit sphere? In one dimension such examples abound; $f(z) = z$ will do the job.

The book of Koosis gives an introduction to the one dimensional theory of H_p spaces. A good way to see what techniques must be developed in such a book comes from looking at three sample theorems proved within the past twenty years.

1. *The maximal ideal space of H_∞ is the closure* (*in the Gelfand topology*) *of* Δ.

2. $(\mathrm{Re}\, H_1(0))^* = BMO$.

3. *The unit ball of H_∞ is the norm closed convex hull of the Blaschke products.*