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ORDINARY $RO(G)$-GRADED COHOMOLOGY

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Let G be a compact Lie group. What is the appropriate generalization of singular cohomology to the category of G-spaces X? The simplest choice is the ordinary cohomology of $EG \times_G X$, where EG is the total space of a universal principal G-bundle. This Borel cohomology [1] is readily computable and has many applications, but is clearly inadequate for such basic parts of G-homotopy theory as obstruction theory. Another choice is Bredon cohomology [2], as generalized from finite to compact Lie groups by several authors. This gives groups $H^n_G(X; M)$ for $n \geqslant 0$ and for a "coefficient system" M. Here M is a contravariant functor from the homotopy category of orbit spaces G/H and G-maps to the category Ab of Abelian groups. (Subgroups are understood to be closed.) Bredon cohomology is adequate for obstruction theory. For finite G, Triantafillou has used it to algebraicize rational G-homotopy theory [11], and she and two of us have used it to set up the foundations of the theory of localization of G-spaces for general G [8]. When M is constant at an Abelian group A, written $M = \mathbf{A}$, we have

$$(*)\qquad H^n_G(X; \mathbf{A}) = H^n(X/G; A).$$

In particular, we have

$$H^n_G(EG \times X; \mathbf{A}) = H^n(EG \times_G X; A).$$

Thus Bredon cohomology generalizes Borel cohomology.

Nevertheless, we feel that these theories do not comprise the full equivariant generalization of ordinary singular cohomology. The full theory should build in the interplay relating the Burnside ring $A(G)$, the real representation ring $RO(G)$, and G-homotopy theory. Any cohomology theory must be "stable". Bredon cohomology is only stable in the classical sense that

$$\widetilde{H}^n_G(X; M) = \widetilde{H}^{n+q}_G(\Sigma^q X; M)$$

for a based G-space X (with basepoint a fixed point). A fully equivariant theory E^* should allow groups $E^v(X)$ for all G-representations V; $E^n(X)$ should be the special case of the trivial representation R^n. If SV denotes the 1-point compactification of V and $\Sigma^v X$ denotes $X \wedge SV$, we should have

$$\widetilde{E}^v(X) = \widetilde{E}^{v \oplus w}(\Sigma^w X).$$

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