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THE NEUMANN PROBLEM ON LIPSCHITZ DOMAINS

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Let D be a Lipschitz domain in $\mathbf{R}^n$, $n > 2$. Let σ denote surface measure on ∂D, and let $\partial/\partial n$ denote the normal derivative on ∂D. In this note we use an *a priori* estimate due to Payne and Weinberger [6], to bound the nontangential maximal function of the gradient ∇u of a (generalized) solution to the Neumann problem

$$\Delta u = 0 \quad \text{in } D; \qquad \frac{\partial}{\partial n} u = g \quad \text{on } \partial D \tag{1}$$

for boundary data g in $L^2(d\sigma)$. A corollary is that ∇u attains its boundary values nontangentially pointwise almost everywhere and through dominated convergence in L^2 on level sets that tend to ∂D. Moreover, u belongs to the Sobolev space $H_{3/2}(D)$. We obtain the same bound and corollary when u is the solution to the Dirichlet problem

$$\Delta u = 0 \quad \text{in } D; \qquad u = f \quad \text{on } \partial D,$$

where f and its gradient on ∂D belong to $L^2(d\sigma)$. For C^1 domains, these estimates were obtained by A. P. Calderón et al. [1]. For dimension 2, see (d) below.

In [4] and [5] we found an elementary integral formula (7) and used it to prove a theorem of Dahlberg (Theorem 1) on Lipschitz domains. Unknown to us, this formula had already been discovered long ago by Payne and Weinberger and applied to the Dirichlet problem in smooth domains. Moreover, they used a second formula (2), which is a variant of a formula due to F. Rellich [7], to study the Neumann problem in smooth domains. We show here that the same strategy as in [4] applied to the second formula (2) coupled with Dahlberg's theorem yields our main result. Thus integral formulas give appropriate estimates for the solution of not only the Dirichlet problem, but also the Neumann problem on Lipschitz domains. We will present a more general version that applies to variable coefficient operators, systems, and other elliptic problems in a later

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