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OPEN ACYCLIC 3-MANIFOLDS, A LOOP THEOREM AND THE POINCARÉ CONJECTURE[1]

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In [3] the 3-dimensional Poincaré conjecture (hereafter denoted P. C.) was reduced to certain questions about open, irreducible, acyclic 3-manifolds. That reduction is strengthened here.

THEOREM 1. *P. C. iff every open, irreducible, acyclic* 3-*manifold, which is the degree one proper image of an open* 3-*manifold embeddable in* S^3, *is also embeddable in* S^3.

DEFINITIONS. A 3-manifold, U^3, is *irreducible* iff every 2-sphere, P.L. embedded in U^3, bounds a 3-ball in U^3. A map is *proper* iff the preimage of any compactum is a compactum.

Let f: $V^3 \longrightarrow U^3$ be a proper map of open, orientable 3-manifolds (not necessarily connected) and M^3 a compact 3-submanifold of V^3 such that f is transverse to M^3. Denote $N^3 = f^{-1}(M^3)$. Then $f|_{N^3}$: $(N^3, \partial N^3) \longrightarrow (M^3, \partial M^3)$. Furthermore if U^3 and V^3 are oriented their orientations induce orientations on M^3 and N^3. Corresponding to these orientations we have elements $\alpha_M \in H_3(M^3, \partial M^3)$ and $\beta_N \in H_3(N^3, \partial N^3)$. f is *degree one* iff there exist orientations of U^3 and V^3 such that for every such M and N as above $f_*(\beta_N) = \alpha_M$. Given such an f we say U^3 is a *proper degree one image* of V^3. A space is *acyclic* iff its first homology group with $\mathbf{Z}$ coefficients is trivial. A noncompact 3-manifold, P^3, is *acyclic at* ∞ iff given any compact subset, X, of P^3 there exists a compact subset, Y, of P^3 such that $X \subset Y$ and i_*: $H_1(P^3 - Y) \longrightarrow H_1(P^3 - X)$ is trivial (where i: $P^3 - Y \longrightarrow P^3 - X$ is inclusion).

Note. Acyclic implies acyclic at ∞.

A *virtual disk* is a space homeomorphic to $D^2 - X$ where X is a (generally nonpolyhedral) compact subspace of $\mathring{D}^2$ (where D^2 is the 2-disk). A proper map f: $\check{D} \longrightarrow U^3$, of a virtual disk $\check{D}$, is a *virtual disk in* U^3 (referred to as a virtual disk if the range is already specified). If f is an embedding then it is an *embedded virtual disk.*

NOTATION. Given α: $S^1 \longrightarrow X$, $[\alpha]$ will denote the conjugacy class of $\pi_1(X)$ determined by α.

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