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ON A CONJECTURE OF PAPAKYRIAKOPOULOS

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ABSTRACT. We disprove a conjecture of Swarup which in turn disproves a well-known conjecture of Papakyriakopoulos that a certain cover is planar.

Let

$$K_n = \left\langle a_1, b_1, \ldots, a_n, b_n; \prod_{i=1}^{n} (a_i, b_i) \right\rangle$$

and

$$J_n = \left\langle a_1, b_1, \ldots, a_n, b_n; \prod_{i=1}^{n} (a_i, b_i), (a_1, b_1\tau) \right\rangle,$$

where n is a fixed integer $\geqslant 2$ and τ is an element of the commutator subgroup of the free group $F(\{a_1, b_1, \ldots, a_n, b_n\})$. Further let S_n be the orientable closed surface of genus n. The fundamental group of S_n is K_n. Papakyriakopoulos [3] put forward the following

P.1. CONJECTURE. (a) J_n is torsion free and

(b) the cover of S_n corresponding to the kernel of the natural group homomorphism $K_n \longrightarrow J_n$ is planar.

Papakyriakopoulos [3] showed that if P.1. is true, then so is the Poincaré Conjecture.

G. A. Swarup [5] has posed the following

P.2. CONJECTURE. The group J_n is a nontrivial free product.

G. A. Swarup [5] showed that

$$\text{P.1.} \Rightarrow \text{P.2.} \Rightarrow \text{Poincaré Conjecture.}$$

THEOREM. *The conjecture* P.2. *is not in general true. Hence the conjecture* P.1. *is not in general true.*

PROOF. Let $G_1 = \langle a_1, b_1; (a_1, b_1 c)\rangle$, where c is any fixed element of the commutator subgroup $F(\{a_1, b_1\})'$ of the free group $F(\{a_1, b_1\})$ so that $(a_1, b_1 c)$ is not conjugate to $(a_1, b_1)^{\pm 1}$ in $F(\{a_1, b_1\})$. For example one could take

$$c = (a_1, b_1).$$

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