

RESEARCH ANNOUNCEMENTS

THE TETRAGONAL CONSTRUCTION

BY RON DONAGI¹

1. Preliminaries. Let C be a nonsingular curve of genus g , and $\pi: \tilde{C} \rightarrow C$ an unramified double cover. The Prym variety $\mathcal{P}(C, \tilde{C})$ is by definition $\ker^0(Nm)$, where $Nm: J(\tilde{C}) \rightarrow J(C)$ is the norm map, and \ker^0 is the connected component of 0 in the kernel. By [M] this is a $(g - 1)$ -dimensional, principally polarized abelian variety. Let A_g, M_g, R_g denote, respectively, the moduli spaces of g -dimensional principally polarized abelian varieties, curves of genus g , and pairs (C, \tilde{C}) as above. (R_g is a $(2^{2g} - 1)$ -sheeted cover of M_g .) The Prym map is the morphism

$$P = P_g: R_g \rightarrow A_{g-1}, \quad (C, \tilde{C}) \mapsto \mathcal{P}(C, \tilde{C}).$$

It is analogous to the Jacobi map $J = J_g: M_g \rightarrow A_g$ sending a curve to its Jacobian. The main reason for studying P is that its image in A_{g-1} is larger than that of J , hence it allows us to handle geometrically a wider class of abelian varieties than just Jacobians. For instance, P_g is dominant for $g \leq 6$ [W] while J_g is only dominant for $g \leq 3$.

The purpose of this announcement is to describe the fibers of P in the various genera. Our main tool for this is a simple-minded construction which we describe in some detail in paragraph 6. Let us use “ n -gonal” (trigonal, tetragonal, etc.) to describe a pair (C, f) where $f: C \rightarrow \mathbf{P}^1$ is a branched cover of degree n (3, 4 respectively). Briefly, our construction takes the data (C, \tilde{C}, f) where $(C, \tilde{C}) \in R_g$ and (C, f) is tetragonal, and returns two new sets of data, (C_0, \tilde{C}_0, f_0) and (C_1, \tilde{C}_1, f_1) , of the same type. This procedure is symmetric: starting with (C_0, \tilde{C}_0, f_0) we end up with (C, \tilde{C}, f) and (C_1, \tilde{C}_1, f_1) . It is useful due to the following observation.

PROPOSITION 1.1. *The tetragonal construction commutes with the Prym map:*

$$\mathcal{P}(C, \tilde{C}) \approx \mathcal{P}(C_0, \tilde{C}_0) \approx \mathcal{P}(C_1, \tilde{C}_1).$$

Received by the editors September 3, 1980,
 1980 *Mathematics Subject Classification.* Primary 14H15, 14H30, 14K10, 32G20,
 14H40.

¹Research partially supported by NSF Grant MCS #77-03876.

© 1981 American Mathematical Society
 0002-9904/81/0000-0103/\$03.25