BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 4, Number 2, March 1981

CANONICAL MAPPINGS BETWEEN TEICHMÜLLER SPACES[1]

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Introduction. In an important survey article **[B10]** Bers reported on the state of knowledge of Teichmüller theory. There has been a lot of progress in the field since that time. The purpose of this paper is to summarize the recent work in one area of Teichmüller space theory. We will concentrate on the hyperbolic properties of Teichmüller spaces, and present as many consequences of this hyperbolicity as we can. The starting point of this study is Royden's **[Ro]** fundamental paper showing that the Teichmüller metric on $T(p, 0)$, $p \geqslant 2$, and the hyperbolic (Kobayashi **[Ko**, pp. 45–46]) metric are one and the same. The organization of the material of this paper follows that of an earlier joint paper with Clifford Earle **[EK1]**, except that an introductory section on history and motivation has been added.

We have neglected completely another area of Teichmüller space theory in which a tremendous amount of recent work has contributed greatly to our understanding of Riemann surfaces; namely, the study of fibrations and boundaries of Teichmüller spaces. I will only mention the people who have contributed to developments in this area: Abikoff, Bers, Earle, Hubbard, Jørgensen, Kerckhoff, Marden, Maskit, Masur, Thurston.

I am grateful to Bernard Maskit for reading an earlier version of this manuscript and for his many helpful suggestions regarding this article, in particular, and Kleinian groups, in general; and to Lipman Bers for his continual help, encouragement, and interest in all aspects of mathematics.

0. A short history and the most classical example.

0.1. The classical theory of moduli of Riemann surfaces originated with Riemann's observation that the conformal type of a compact Riemann surface of genus $p > 1$ depends on $3(p - 1)$ complex parameters, known as moduli. Yet, the fact that the *space of moduli of compact surfaces of genus* $p > 1$ forms a normal complex space was not proven until the early 1950's. The key step is passing to a cover of the space of moduli, known as the *Teichmüller space of genus p*, $T(p, 0)$. The space $T(p, 0)$ appears implicitly in the continuity arguments of Klein and Poincaré. It was constructed explicitly by Fricke **[FK]** and Fenchel-Nielsen **[FN]**. Fricke proved that $T(p, 0)$ is a

This article is an expanded version of an Invited Address delivered at the 769th meeting of the American Mathematical Society held at Syracuse, New York on October 28, 1978; received by the editors July 15, 1980.

1980 *Mathematics Subject Classification*. Primary 30–02; Secondary 32G15, 32H20, 30C60, 30F10, 30F35.

[1]Research partially supported by NSF Grant MCS7801248.