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GEOMETRY AND PROBABILITY IN BANACH SPACES

BY LAURENT SCHWARTZ

Introduction. The following is a brief survey of results in a circle of ideas concerned with the properties of various classes of Banach spaces (centering around the classical L^p spaces) in terms of operators acting between them, a circle of ideas and important results which involves the names of such mathematicians as Pietsch, Maurey, H. Rosenthal, Krivine, Pisier, and others. As the title emphasizes, an important role is played by random processes with values in these Banach spaces, i.e. vector-valued probability distributions.

1. Summing maps in Banach spaces. A sequence $e = (e_n)_{n\in\mathbf{N}}$ of elements of a Banach space E is said to be l^p if $\|e\|_p = (\Sigma_n \|e_n\|_E^p)^{1/p} < +\infty$; $\|e\|_p$ is the p-norm of this sequence (in fact, it is a norm only for $1 \leqslant p < +\infty$; we shall also work in the case $0 < p \leqslant +\infty$, with the usual modifications for $p = +\infty$: $\|e\|_{+\infty} = \operatorname{Sup}_n \|e_n\|_E$). A sequence e is said to be scalarly l^p if, for every $\xi \in E'$ (the dual of E), the scalar sequence $\langle e, \xi\rangle = (\langle e_n, \xi\rangle)_{n\in\mathbf{N}}$ is l^p, that is $(\Sigma_n |\langle e_n, \xi\rangle|^p)^{1/p} < +\infty$; in this case, it can be proved (by the Banach-Steinhaus theorem or the closed graph theorem) that $\|e\|_p^* = \operatorname{Sup}_{\|\xi\| \leqslant 1} (\Sigma_n |\langle e_n, \xi\rangle|^p)^{1/p} < +\infty$; $\|e\|_p^*$ is the scalarly l^p-norm of e. A continuous linear map u from a Banach space E into a Banach space F transforms trivially an l^p-sequence into an l^p-sequence, a scalarly l^p-sequence into a scalarly l^p-sequence; u is said to be p-summing if it transforms every scalarly l^p-sequence into an l^p-sequence. By a trivial argument, if u is p-summing, there exists a constant C such that, for every sequence e of E, the inequality $\|u(e)\|_p \leqslant C\|e\|_p^{\#}$ holds; the smallest constant C is called the p-summing norm of u and is denoted $\pi_p(u)$.

Every map is $(+\infty)$-summing, since $\|e\|_{+\infty}^* = \|e\|_{+\infty}$, and $\pi_{+\infty}(u) = \|u\|$; generally u will not be better (for instance, we shall see that the identity map in an infinite-dimensional Banach space is never p-summing for $p < +\infty$). On the other hand, if E is finite dimensional, a scalarly l^p-sequence is also l^p, so that every map u of finite rank is p-summing for every p. A finite sum of p-summing maps is p-summing; a finite product of continuous linear maps, one of which is p-summing, is also p-summing (the p-summing maps "form an ideal").

THEOREM (1.1) (PIETSCH). *For u: $E \to F$ to be p-summing, $p < +\infty$, it is necessary and sufficient that there exists a Radon probability μ on the unit disk*

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