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A POINCARÉ-HOPF TYPE THEOREM FOR THE DE RHAM INVARIANT

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The Poincaré-Hopf theorem relates the Euler-characteristic of a manifold to the local behavior of a generic vector-field on the manifold in a neighborhood of its zeroes. As a corollary of this, by taking the gradient, one can calculate the Euler-characteristic of a manifold from a local knowledge of a generic map to R^1 around its singular points. We prove an analogue of this theorem for calculation of the de Rham invariant of $4k + 1$ dimensional orientable manifolds from a map to R^2.

For $4k + 1$ dimensional orientable manifolds we have the de Rham invariant $d(m)$. This invariant is

(a) the rank of the 2-torsion in $H_{2k}(M)$,

(b) $\hat{\chi}_Q(M) - \hat{\chi}_2(M)$ mod 2 where $\hat{\chi}_F(M)$ is the semicharacteristic of M with coefficients in F,

(c) $d(M) = [w_2 w_{4k-1}(M), [M]] = [v_{2k} sq^1 v_{2k}(M), [M]]$,

where $w_i(M)$ is the ith Stiefel-Whitney class and v_i is the ith *Wu* class of M.

For the equivalence of these definitions see [L-M-P]. The de Rham invariant is important in the theory of surgery; see [M] or [M-S].

Definition of the local invariant. Let M^m, N^n be C^∞ manifolds. Let $C^\infty(M, N)$ be the space of C^∞ maps from M to N topologized with the C^∞ topology. Within $C^\infty(M, N)$ we have a dense (in fact residual) subset $G(M, N)$ of maps which are generic in the sense of Thom-Boardman [B] and satisfy the normal crossing condition [G-G]. This second condition is essentially that f is in general position as a map of its singularity submanifolds to N.

Let $f \in G(M, R^2)$; then df is of rank 2 except on a collection of disjoint closed curves in M, the singular set of f, $S_1(f)$. At points of $S_1(f)$, df is of rank 1. Restricted to $S_1(F)$ f is an immersion except at a finite set of points, $S_{1,1}(f)$, the cusp points of f. $S_1(f) - S_{1,1}(f) = S_{1,0}(f)$ is the set of fold points of f. Suppose $x \in S_{1,0}(f)$ then we can choose coordinates $x_1, \ldots, x_n$ around x and coordinates y_1, y_2 around $f(x)$ so that

$$f(x_1, \ldots, x_n) = (x_1, x_2^2 + x_3^2 + \cdots + x_k^2 - x_{k+1}^2 - \cdots - x_n^2).$$

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