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GEOMETRIC RELATIONS BETWEEN HOMEOMORPHIC RIEMANN SURFACES

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In complex analysis one often investigates a particular class of functions on some domain. More often than not the domain is simply connected and if it is not the whole plane, it is usually taken to be the unit disk. For this is conformally equivalent to any other nondegenerate simply connected region and the conformal map induces a certain isomorphism of the given class of functions to a class on the disk. What doesn't go over nicely, in general, are boundary properties of functions.

On the other hand if the domain is multiply connected, it is no longer true that any two nondegenerate ones of the same connectivity are conformally equivalent. Thus before we can start to examine deeper relations between corresponding classes of functions, we must understand how the domains themselves are related.

Teichmüller space is a space of domains all of the same topological type. In that theory we find out how the domains are related to each other and then parametrize them. At least this is true for domains of finite connectivity. Once we do this, we can understand, for example, how the canonical domain functions like the harmonic measures vary real analytically in the parameters.

The foundation of Teichmüller space theory is thus rooted in methods for comparing two domains of the same topological type. The theory is most completely worked out and in any case has the nicest expression when the domains involved have no boundary at all; that is are compact Riemann surfaces without boundary. For this reason our explanations will ultimately be restricted to this case.

We start with two Riemann surfaces R and S and an (orientation preserving) homeomorphism f: $R \to S$. Since we are really going to deal with conformal equivalence classes, if f is homotopic to a conformal map we consider that S is the same as R and f is the identity. Of course given two surfaces R, S, there are in general infinitely many choices for homeomorphisms f, no two being homotopic. For example, here are two; the image regions are the same in both cases but the two images of the arc between punctures lie in different homotopy classes.

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