

MARSTON MORSE AND HIS MATHEMATICAL WORKS

BY RAOUL BOTT¹

1. Introduction. Marston Morse was born in 1892, so that he was 33 years old when in 1925 his paper *Relations between the critical points of a real-valued function of n independent variables* appeared in the Transactions of the American Mathematical Society. Thus Morse grew to maturity just at the time when the subject of Analysis Situs was being shaped by such masters² as Poincaré, Veblen, L. E. J. Brouwer, G. D. Birkhoff, Lefschetz and Alexander, and it was Morse's genius and destiny to discover one of the most beautiful and far-reaching relations between this fledgling and Analysis; a relation which is now known as *Morse Theory*.

In retrospect all great ideas take on a certain simplicity and inevitability, partly because they shape the whole subsequent development of the subject. And so to us, today, Morse Theory seems natural and inevitable. However one only has to glance at these early papers to see what a tour de force it was in the 1920's to go from the mini-max principle of Birkhoff to the Morse inequalities, let alone extend these inequalities to function spaces, so that by the early 30's Morse could establish the theorem that for any Riemann structure on the n -sphere, there must be an infinite number of geodesics joining any two points.

This whole flight of ideas was of course acclaimed by the mathematical world. It brought him to the Institute for Advanced Study in 1935, when, at 43, he also delivered the Colloquium Lectures of the Mathematical Society and wrote his monumental book on the Calculus of Variations in the Large; it eventually earned him practically every honor of the mathematical community, over twenty honorary degrees, the National Science Medal, the Legion of Honor of France,

Nevertheless, when I first met Marston in 1949 he was in a sense a solitary figure, battling the *algebraic topology*, into which his beloved Analysis Situs had grown. For Marston always saw topology from the side of Analysis, Mechanics, and Differential geometry. The unsolved problems he proposed had to do with dynamics—the three body problem, the billiard ball problem, and so on. The development of the algebraic tools of topology, or the project of bringing order into the vast number of homology theories which had sprung up in the thirties—and which was eventually accomplished by the Eilenberg-Steenrod axioms—these had little interest for him. "*The battle between algebra and geometry has been waged from antiquity to the present*" he wrote in his address *Mathematics and the Arts* at Kenyon College in 1949, and

Received by the editors April 15, 1980.

¹This work was supported in part through funds provided by the National Science Foundation under the grant 33-966-7566-2.

²Poincaré was born in 1854, the others all in the 1880's.

© 1980 American Mathematical Society
0002-9904/80/0000-0500/\$12.00