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TRIPLE COLLISIONS IN THE ISOSCELES 3-BODY PROBLEM

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We consider here the plane 3-body problem of celestial mechanics forming an isosceles configuration at all times. We first study the topology of the energy submanifolds E_h with the triple collision manifold T (Mc Gehee [4]) as an added boundary, which corresponds to blowing up the collision. By a time transformation we scale the vectorfield extending it to the boundary. We then analyze the fictitious flow on T to get information about the actual neighboring flow on E_h. Our results are akin to Mc Gehee's for the collinear case; but the fictitious flow is more interesting here, having 6 instead of 2 critical points.

Devaney [1] has simultaneously studied the flow of this problem. There is considerable overlap of his paper with the results we state after Theorem 1, except for his last section as explained at the end of this announcement. In [7] Simó has recently described the flow more completely.

To get such an isosceles motion, the two masses at the symmetrical vertices must be equal, with a fixed symmetry axis, about which the initial velocities must be balanced.

Denote by μ the equal masses and by m the third one, introducing Jacobi-coordinates [5]: $x \geqslant 0$ is the semidistance between the equal masses, and $y \in R$ is the signed distance from m to the segment joining the others. Dividing out by a 2μ factor, we take the simplified lagrangian

$$L(x, y, \dot{x}, \dot{y}) = (\dot{x}^2 + \dot{y}^2/\alpha^2)/2 + U(x, y), \tag{1}$$

where $\alpha^2 = 1 + 2\mu/m$, and the potential function is

$$U(x, y) = \mu/(4x) + m/\sqrt{x^2 + y^2}. \tag{2}$$

The associated hamiltonian (total energy) is

$$E(x, y, p_1, p_2) = (p_1^2 + \alpha^2 p_2^2)/2 - U(x, y). \tag{3}$$

If we set $q = (x, y)$, $p = (p_1, p_2)$ and define a matrix $M = \operatorname{diag}(1, \alpha^{-2})$, the Hamilton equations for (3) can be writen in the familiar form

$$\dot{q} = M^{-1}p, \qquad \dot{p} = \operatorname{grad} U(q),$$

and the energy relation $E = h$, defining any energy surface can be written as

$$\tfrac{1}{2}M^{-1}p^2 = U(q) + h.$$

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