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$K_r(\mathbf{Z}/p^2)$ AND $K_r(\mathbf{Z}/p[\epsilon])$ FOR $p \geqslant 5$ AND $r \leqslant 4$

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If R is a ring, $K_0(R)$ is the Grothendieck group of finitely generated projective R-modules, $K_1(R)$ is the abelianization of the group $GL(R)$ of invertible matrices over R, and $K_2(R)$ is the second homology group of $E(R) = \ker(GL(R) \longrightarrow K_1(R))$. Higher K-groups are defined as homotopy groups of a space associated to $GL(R)$ and provide additional homological invariants of the linear algebra of R. Unfortunately, these higher (degree greater than 2) K-groups appear difficult to compute even for very simple rings: in particular, no higher K-groups of rings with nilpotents have been computed. We present computations for two such rings, $\mathbf{Z}/p^2\mathbf{Z}$ and $\mathbf{Z}/p[\epsilon]$ (the dual numbers over $\mathbf{Z}/p$).

Before stating our results, we briefly mention other computations of higher K-groups. Quillen [9] computed $K_i(\mathbf{F}_q)$ for any $i \geqslant 0$ and any finite field $\mathbf{F}_q$. Browder [3], Harris and Segal [6], Quillen [11], and Soule [12] have partial results on higher K-groups of rings of integers in number fields. Borel [2] has computed the ranks of the K-groups of such rings. Lee and Szczarba [7] have computed $K_3(\mathbf{Z})$. Moreover, Quillen [10] has proved many general theorems which enable one to convert known computations of various rings to computations of related rings.

We announce the following theorems whose proofs will appear in [5].

THEOREM 1. *Let $p \geqslant 5$ be a prime. Let $\mathbf{Z}/p[\epsilon]$ denote the ring (of order p^2) of dual numbers over $\mathbf{Z}/p$.*

$$K_1(\mathbf{Z}/p^2) = K_1(\mathbf{Z}/p[\epsilon]) = \mathbf{Z}/p - 1 \oplus \mathbf{Z}/p,$$

$$K_2(\mathbf{Z}/p^2) = K_2(\mathbf{Z}/p[\epsilon]) = 0,$$

$$K_3(\mathbf{Z}/p^2) = \mathbf{Z}/p^2 - 1 \oplus \mathbf{Z}/p^2; K_3(\mathbf{Z}/p[\epsilon]) = \mathbf{Z}/p^2 - 1 \oplus \mathbf{Z}/p \oplus \mathbf{Z}/p,$$

$$K_4(\mathbf{Z}/p^2) = K_4(\mathbf{Z}/p[\epsilon]) = 0.$$

Of course, $K_1(\mathbf{Z}/p^2)$ and $K_1(\mathbf{Z}/p[\epsilon])$ are well known [1, V. 9.1], $K_2(\mathbf{Z}/p^2)$ was computed by Milnor [8], and $K_2(\mathbf{Z}/p[\epsilon])$ was computed by van der Kallen [13].

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