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ON HURWITZ' "$84(g-1)$ THEOREM" AND PSEUDOFREE ACTIONS

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1. Definitions. An *admissible* space is a finitistic space cf. **[B**, p. 133**]** with finitely generated integral homology. Let a group G act on a topological space X, and $\pi: X \longrightarrow G\backslash X$ the corresponding projection onto the orbit space. Then $S = \bigcup_{g \in G-e} X^g$ is the *singular* set and $\pi(S)$ the *branch* set of the G-action. The G-action is said to be *free* resp. *semifree* resp. *pseudofree* if $S = \emptyset$ resp. $S = X^G$ resp. S is discrete. If X is admissible, G finite acting pseudofreely on X then S is finite. In this case for $P^* \in \pi(S)$ the order n_{p^*} of the isotropy subgroup of G at $P \in \pi^{-1}(P^*)$ is called the *branching index* of the action at P^*.

2. An extension of Hurwitz' theorem. Let Aut X denote the full group of automorphisms of a Riemann surface X. If X is closed and has genus $g \geqslant 2$ Hurwitz cf. **[H]** proved that $|\mathrm{Aut}\, X| \leqslant 84(g-1)$ and that the action of Aut X on the space of holomorphic differentials is faithful. Now every nonidentity holomorphic selfmap of X has only *isolated* fixed points and also Aut X (since it leaves a Riemannian metric invariant) is a compact Lie group. This theorem then extends to arbitrary pseudofree actions as follows. Let $\mathbf{N} = \{1, 2, \cdots\}$ and χ denote the Euler characteristic.

(2.1) THEOREM. *There exists $h: \mathbf{N} \longrightarrow \mathbf{R}_{>0}$ with the following property. If X is admissible, $\chi(X) < 0$, $m(X) = \Sigma_{i \geqslant 0} \dim H_{2i}(X; Q)$ and G is a compact Lie group acting on X so that every finite subgroup acts pseudofreely. Then G is finite, $|G| \leqslant h(m(X))|\chi(X)|$ and the action of G on $H_*(X; Q)$ is faithful.*

EXAMPLES. One has $h(1) = 6$, $h(2) = 42$, $h(3) = 1806, \cdots$. In Hurwitz' theorem one has $m(X) = 2$, $\chi(X) = 2 - 2g$ and so $h(2)|\chi(X)| = 84(g-1)$. For all other closed or nonclosed surfaces with $\chi(X) < 0$ one has $m(X) = 1$ and the bound for G is $6|\chi(X)|$. In the case of a closed nonorientable surface U_h with $h \geqslant 3$ crosscaps this bound is $6(h-2)$. As in the classical case cf. **[M]**, **[S]** it may be shown that this bound is attained for infinitely many h's. Concretely there exist pseudofree actions of the alternating groups A_4, A_5 and the

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