BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 2, Number 2, March 1980

SCATTERING THEORY FOR AUTOMORPHIC FUNCTIONS

BY PETER D. LAX AND RALPH S. PHILLIPS[1]

ABSTRACT. This paper is an expository account of our 1976 monograph [6] on *Scattering theory for automorphic functions*. Several improvements have been incorporated: a more direct proof of the meromorphic character of the Eisenstein series, an explicit formula for the translation representations and a simpler derivation of the spectral representations. Our hyperbolic approach to the Selberg trace formula is also included.

1. Introduction. In 1972 Faddeev and Pavlov [2] discovered a revealing connection between the harmonic analysis of functions automorphic with respect to a discrete subgroup of $SL(2, R)$ and the Lax-Phillips scattering theory as applied to the non-Euclidean wave equation. Their work is based on the spectral theory for the Laplace-Beltrami operator previously developed by Faddeev [1] using elliptic arguments. In our 1976 monograph [6] we redid the Faddeev-Pavlov paper entirely within the framework of our theory, basing our development on the non-Euclidean wave equation. We obtained new treatments for (i) the spectral theory of the Laplace-Beltrami operator over noncompact domains of finite area; (ii) the meromorphic character of the Eisenstein series over the whole complex plane; and (iii) a new form of the Selberg trace formula.

In this paper we sketch a revised version of our monograph including a more direct proof of the meromorphic character of the Eisenstein series, an explicit formula for the translation representations and a simpler derivation of the spectral representations.

The harmonic analysis of automorphic functions has been extensively studied; references to the pertinent parts of this theory are contained in our monograph. We recall that the Poincaré plane Π, that is the upper half plane

$$w = x + iy, \qquad y > 0, \tag{1.1}$$

serves as a model for a non-Euclidean geometry in which the motions are given by the group G of fractional linear transformations:

$$w \to \frac{aw + b}{cw + d} \tag{1.2}$$

where a, b, c, d are real and

$$ad - bc = 1; \tag{1.3}$$

Received by the editors August 1, 1979.
AMS (*MOS*) *subject classifications* (1970). Primary

[1]The work of both authors was supported in part by the National Science Foundation, the first author under Grant No. MCS-76-07039 and the second under Grant No. MCS-77-04908 A 01.

0002-9904/80/0000-0100/$09.75