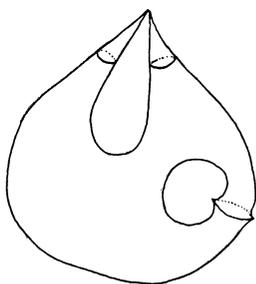


A TOPOLOGICAL CHARACTERIZATION OF REAL ALGEBRAIC VARIETIES

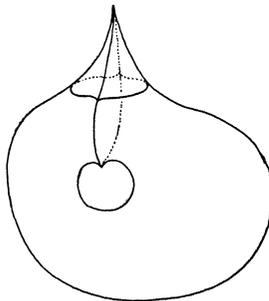
BY SELMAN AKBULUT AND HENRY C. KING

We show that if a smooth locally conelike stratified set admits a certain kind of topological resolution then it is homeomorphic to a real algebraic set, i.e. zeros of polynomial functions (this generalizes [AK₁], [AK₂]). We expect that the algebraic resolution of singularities [H] implies that every algebraic set admits such a topological resolution, hence it is reasonable to suspect that we have a complete topological characterization of real algebraic sets.

Examples of some stratified sets admitting such a topological resolution are spaces which we call A_k -spaces, $k = 0, 1, 2, \dots$. We define A_k -spaces inductively by saying that A_0 -spaces are smooth compact manifolds, and an A_k -space is a compact smooth stratified set X with a trivialization of a neighborhood of each stratum X_i , $h_i: X_i \times \text{cone}(\Sigma_i) \rightarrow X$ where Σ_i is an A_{k-1} -space which bounds a compact A_{k-1} -space with boundary (h_i required to be compatible with the trivializations of neighborhoods of the strata of Σ_i).



an A_1 -space



an A_2 -space

The topological resolution of an A_k -space X is obtained by a sequence of 'blow ups' as follows: take a lowest dimensional stratum X_i (the 'center' of the 'blow ups') with trivialization $h_i: X_i \times \text{cone}(\Sigma_i) \rightarrow X$ and replace $h_i(X_i \times \text{cone}(\Sigma_i))$ by $X_i \times W_i$ where W_i is a compact A_{k-1} -space which Σ_i

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