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AN ANALOGUE OF THE MOSTOW-MARGULIS RIGIDITY THEOREMS FOR ERGODIC ACTIONS OF SEMISIMPLE LIE GROUPS[1]

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In this paper we announce an analogue of the rigidity theorems for lattices in semisimple Lie groups of Mostow and Margulis in the context of ergodic actions of semisimple Lie groups and ergodic foliations by symmetric spaces.

If G, G' are locally compact (second countable) groups acting ergodically on standard measure spaces S, S' respectively, the actions are called orbit equivalent if, possibly after discarding invariant null sets, there is a measure class preserving Borel bijection $S \longrightarrow S'$ that takes orbits onto orbits. For example, if $G = G'$ and the actions are conjugate (i.e., the map $S \longrightarrow S'$ is a G-map), or more generally, automorphically conjugate (i.e., conjugate modulo an automorphism of the group) then the actions are clearly orbit equivalent. However, orbit equivalence is in general a far weaker notion. For example, if G and G' are discrete amenable groups and the measures are finite and invariant (and properly ergodic, i.e., not transitive modulo a null set), then the actions will be orbit equivalent [1], [2], [7]. The same result is true for essentially free (i.e., almost all stabilizers are trivial) actions of continuous amenable groups. Our main result is Theorem A which asserts that the situation for semisimple Lie group actions is very different. This result has been independently conjectured by A. Connes. An ergodic action of a semisimple Lie group is called irreducible if the restriction to every noncentral normal subgroup is still ergodic.

THEOREM A. *Let G, G' be connected seimsimple Lie groups with finite center and no compact factors, and suppose the real rank of G is at least* 2. *Suppose S, S' are essentially free, irreducible ergodic G, G' spaces respectively with finite invariant measure. If the actions are orbit equivalent then G and G' are locally isomorphic. If G and G' have trivial center, then $G = G'$ and the actions are automorphically conjugate.*

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