

toposes in place of Boolean-valued models. So far this has been done (by M. Bunge; see p. 329) only for the independence of the Souslin hypothesis.

These are by no means the only connections with logic. Deligne's theorem, that every "coherent" topos has enough "points" (Theorem 7.44), is intimately related to the Gödel-Henkin completeness theorem for finitary first order theories, and there is (Theorem 7.16) a similar categorical version of the Lowenheim-Skolem theorem. In other words, topos theory not only developed from a collision of algebraic geometry and set theory, but this collision has set off various other surprises: Sheaves appearing in set theory and completeness theorems in algebraic geometry. Other connections—with cohomology theory, with torsors, and with profinite fundamental groups—are left for the reader to discover in Johnstone's book.

This book does provide good examples of the better understanding promised in the introduction. To achieve this understanding, the reader must on occasion study hard, to get at what is behind the economical presentation, with little motivation, of all the techniques and corresponding theorems. Only by choosing this austere presentation was the author able to bring all these (and many other ideas) in the brief compass of 360 pages.

There is a very helpful index of notation at the back. Given the range of theorems collected from many authors reported here, usually in neater and quicker ways, I located very few slips; Theorem 0.14 from Eilenberg and Moore is misquoted, while Theorem 7.37(i) from Grothendieck on coherent topoi is misproved; both can be corrected by reference to the original sources. Lemma 9.17 is misnumbered—but enough of such carping comments. This is a dense and rich book, which has organized valuable material as an aid to our deeper understanding of sheaf theory, logic, and algebra.

SAUNDERS MAC LANE

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Symmetry and separation of variables, by Willard Miller, Jr., Addison-Wesley Publishing Company, Reading, Massachusetts, 1977, xxx + 285 pp., \$21.50.

Separation of variables is a technique for solving special partial differential equations. It is taught in elementary courses on partial differential equations, but the method usually does not achieve the status of a mathematical theory.

Because most references do not give a precise definition of separation of variables, I invented a definition myself. Let us call a partial differential equation in n variables x_1, \dots, x_n *separable* if there are n ordinary differential equations in x_1, \dots, x_n , respectively, jointly depending on $n - 1$ independent parameters (the separation constants), such that, for each choice of the parameters and for each set of solutions (X_1, \dots, X_n) of the o.d.e.'s, the function $u(x_1, \dots, x_n) := X_1(x_1) \cdots X_n(x_n)$ is a solution of the p.d.e. Under the terms of this definition a converse implication often holds: If $u = X_1 \cdots X_n$ is a factorized solution of the p.d.e. then, for some choice of the parameters, the X_i 's are solutions of the o.d.e.'s. The most familiar cases of separability deal with a linear second order p.d.e. which separates into n