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Convexity in the theory of lattice gases, by Robert B. Israel, Princeton Series in Physics, Princeton Univ. Press, Princeton, N. J., 1979, LXXXV + 168 pp., $16.50 (cloth).

Lattice gases? This sounds very much like physics. And what have they to do with convexity? The mathematician may be pardoned if he is puzzled, but he could't do better than to look into this book if he wants to find out what this is all about. Lattice gases are certain mathematical models that occur in statistical mechanics. Statistical mechanics was created a near-century ago by J. Willard Gibbs, who conceived the idea of a general explanation for the laws of thermodynamics. It was also Gibbs who in two pioneering papers, rather neglected ever since, suggested that the proper general formulation of the laws of thermodynamics may be made in terms of certain functions, called thermodynamics potentials, which characterize the physical systems considered, and whose *convexity* is the mathematical expression of the stability of states of thermal equilibrium. Our book under review is actually two books in one; the first is an introductory essay by Arthur Wightman, which contains the historical motivation, an exposition of the Gibbsian ideas, the significance of convexity of the thermodynamic potentials, as well as a brief review of the formalism of statistical mechanics as left to us by Gibbs. This is far more than an introduction, and it alone is worth the price of the book. The reader is advised to come back to it from time to time, when studying the more technical proofs of Israel's chapters, to gain motivation, deepen understanding, and appreciate interconnections.

On to the technicalities. First, definitions. A lattice gas is a mathematical system determined by five things, ν, Ω_0, μ_0, Ω and $\mathfrak{B}$. ν is a positive integer, the "dimension". Ω_0 is a compact Hausdorff space, frequently just a finite set. μ_0 is a distinguished natural normalized measure on Ω_0, e.g. Haar measure if Ω_0 is a group, uniform surface measure if Ω_0 is a sphere, normalized counting measure if Ω_0 is finite. With $\mathbf{Z}$ the set of integers, write $L = \mathbf{Z}^\nu$ (the "lattice"), and think of a copy of (Ω_0, μ_0) attached to each point ("lattice site") of L. Ω is defined as a closed, translation-invariant (under the natural action of the additive group L) subspace of the compact space Ω_0^L; thus a point ("configuration") $\omega \in \Omega$ is a function $L \to \Omega_0$ assigning a "coordinate" $\omega_x \in \Omega_0$ to each $x \in L$. A typical example is $\Omega_0 = \{0, 1\}$, $\Omega = \{\omega \in \Omega_0^L\colon \omega_x\omega_y = 0$