

and Schwartz: Riesz operators, generalized hermitian operators, prespectral (as opposed to spectral) operators, and well-bounded operators. The exposition is well knit, and there are numerous examples. Dowson's book is a fine contribution to the literature, and should benefit both experts and novices. If there is any shortcoming, it would be the absence of exercises.

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Bessel polynomials, by Emil Grosswald, Lecture Notes in Math., vol. 698, Springer-Verlag, Berlin-Heidelberg-New York, 1978, xiv + 182 pp., \$9.80.

Many mathematicians, of whom I am one, find orthogonal polynomials fascinating. I was introduced to the Legendre polynomials by O. D. Kellogg, in a course on potential theory, almost half a century ago. At the time, I was entranced more by their elegant formal properties than by their applications. Later, I encountered other orthogonal polynomials. One of the ways in which they arise is as eigenfunctions of differential equations, where the boundary condition is just that of *being* a polynomial, and so involving only finitely many parameters. Perhaps if high-speed computers had been invented earlier, the computational advantages of polynomial solutions would have seemed less compelling, but it is hard to imagine that the so-called classical polynomials (Laguerre, Hermite, Jacobi–Legendre and Chebyshev are special cases) could have escaped notice for long.

I expect (without having actually investigated their history) that all the named systems had been studied by predecessors of the mathematicians they are named for. The Bessel polynomials, however, are exceptional: they appear not to have been studied by Bessel (although they are related to Bessel functions), and were named by Krall and Frink [2] in 1949. They had, in fact, been more or less known at least since 1873, and had occurred in connection with the irrationality of π , statistics, and the wave equation; and were introduced (independently) at about the same time in electrical engineering. Such an ubiquitous set of polynomials surely deserves not only a name but more than the casual mention it got in the Bateman Project volumes [1] in 1953.

The paper by Krall and Frink was actually the first systematic study of the Bessel polynomials; since objects of mathematical discourse, like continents, are so often named for those who popularize them rather than for those who discover them, it is only because of Krall and Frink's good taste that we do not now know these polynomials as the Krall-Frink polynomials. Some of the subsequent active research on Bessel polynomials seems to have been inspired by Krall and Frink's calling attention to the orthogonality of the polynomials—in the complex plane rather than on the real intervals where the classical polynomials are orthogonal.

Grosswald's bibliography lists 116 titles dealing with Bessel polynomials. The book is a quite detailed survey. It describes not only the analytic properties such as one finds for the classical orthogonal polynomials in Szegő's book [3], but also algebraic properties (irreducibility, the Galois group). Grosswald has also provided abstracts of many results that he could