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RESEARCH ANNOUNCEMENTS

ON THE DIFFERENCE BETWEEN CONSECUTIVE PRIMES

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It was shown by Huxley [1] that

$$\pi(x) - \pi(x-y) \sim \frac{y}{\log x} \qquad (x^{\vartheta} \leqslant y \leqslant \tfrac{1}{2}x), \tag{1}$$

for any constant $\vartheta > 7/12$. It follows that

$$p_{n+1} - p_n \ll p_n^{\vartheta} \tag{2}$$

for $\vartheta > 7/12$, where p_n is the nth prime number. At present the asymptotic formula (1) is not known for any $\vartheta \leqslant 7/12$. However Iwaniec and Jutila [2] have recently shown that, if one asks only for

$$\pi(x) - \pi(x-y) \gg \frac{y}{\log x} \qquad (x^{\vartheta} \leqslant y \leqslant \tfrac{1}{2}x), \tag{3}$$

then $\vartheta \geqslant 13/23$ is admissible. It follows that (2) holds with $\vartheta = 13/23$. Here $7/12 = 0 \cdot 5833 \ldots$, while $13/23 = 0 \cdot 5652 \ldots$. Moreover they indicated that the condition $\vartheta \geqslant 13/23$ could be relaxed to $\vartheta > 5/9 = 0 \cdot 5555 \ldots$, by an elaboration of the argument. The constant 5/9 was the limit of their method.

We can now extend the range of validity of (2) and (3) as follows.

THEOREM. *For any $\vartheta > 11/20$ and $x \geqslant x(\vartheta)$ we have*

$$\pi(x) - \pi(x-y) > \frac{1}{212}\frac{y}{\log x}$$

in the range $x^{\vartheta} \leqslant y \leqslant \tfrac{1}{2}x$. Thus

$$p_{n+1} - p_n \ll p_n^{\vartheta}.$$

Note that $11/20 = 0 \cdot 5500 \ldots$. This constant is the limit of the present method, since $\vartheta > 11/20$ is required in the lemma quoted below.

The proof of our theorem, like that given by Iwaniec and Jutila, uses a combination of the linear sieve and certain weighted zero-density estimates for

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