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# REPRESENTATIONS OF FINITE GROUPS OF LIE TYPE

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The representation theory of a group $G$ over the field of complex numbers involves two problems: first, the construction of the irreducible representations of $G$; and second, the problem of expressing each suitably restricted complex valued function on $G$, as a linear combination (or a limit of linear combinations), of the coefficients of the irreducible representations.

For example, if $G$ is the additive group of real numbers mod 1 (the one-dimensional torus), one considers integrable functions on $G$, or what is the same thing, integrable periodic functions of period 1 on the additive group of real numbers. In this case the irreducible representations of $G$ are given by the exponential functions $x \to e^{2\pi ikx}$, where $k$ is an integer, and are the continuous homomorphisms from $G$ into the multiplicative group of complex numbers. The expression of an integrable function in terms of the irreducible representations $\{e^{2\pi ikx}\}$ is the *Fourier expansion* of $f$,

$$f(x) \sim \sum_{-\infty}^{\infty} a_k e^{2\pi ikx},$$

where the *Fourier coefficients* $a_k$ are given in terms of $f$ and the representations by the formulas

$$a_k = \int_{-\infty}^{\infty} f(t) e^{2\pi ikt}\, dt, \qquad k = 0, \pm 1, \pm 2, \ldots.$$

For such a group, and, more generally, for locally compact groups and Lie groups, the solution of the problems of representation theory involves both the construction of the irreducible representations, and the question of under what circumstances the Fourier expansion converges, and in particular, whether it converges to the function.

The subject of harmonic analysis, to which the preceding considerations lead, is an old and pervasive part of mathematics, pursued for more than a century, and still full of life. It has fostered many triumphs, and certainly one of the greatest has been Harish-Chandra's contributions to harmonic analysis on semisimple Lie groups (see, for example, Warner [I2]).[2]

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[2]References preceded by I are found in the short list at the end of the Introduction; numbered references not preceded by I are at the end of the article.


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