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Geometric asymptotics, by Victor Guillemin and Shlomo Sternberg, Math. Surveys, no. 14, Amer. Math. Soc., Providence, R.I., 1977, xviii + 474 pp., $34.40.

Symplectic geometry and Fourier analysis, by Nolan R. Wallach, Math Sci. Press, Brookline, Massachusetts, 1977, xvii + 436 pp.

1. Introduction. The use of symplectic geometry to describe classical mechanics and to understand it on a deeper level has its origins in the work of Poincaré (1889), Cartan (1922), Siegel (1950) and Reeb (1951). By the 1960s this topic was widely known and was available from several sources such as Mackey [**1963**], Sternberg [**1964**], Abraham [**1967**], Hermann [**1968**] and Godbillon [**1969**].

During the 1960s a new direction and impetus to the field arose when deep links between symplectic geometry, group representations, quantization and linear partial differential equations were found by Keller [**1958**], Segal [**1960**], [**1965**], Kirillov [**1962**], Maslov [**1965**], Egorov [**1969**], Kostant [**1970**], Souriau [**1970**], Hörmander [**1971**] and Duistermaat and Hörmander [**1972**], to mention some of the key contributors. The subject is evolving rapidly and therefore a definitive treatise is not possible at the present time. Nevertheless, the books under review attempt to describe some of these new links.

To penetrate to the basic ideas in either of the books requires extensive background preparation and a large investment of time. However, some of the key ideas are already present in the simplest examples. Therefore we shall spend some time discussing the one dimensional Schrödinger equation and the relation between classical and quantum mechanics. This will give the potential reader a glimpse at the theory and what type of results are obtained.

2. The one-dimensional Schrödinger equation. Let V: $\mathbf{R} \to \mathbf{R}$ be the potential, ψ: $\mathbf{R} \to \mathbf{C}$ the wave function, and let E, $\hbar$, m be constants (energy, Planck's constant and mass, respectively). Consider the stationary Schrödinger equation:

$$L\psi = E\psi$$

where

$$L\psi = -\frac{\hbar^2}{2m}\psi'' + V\psi \tag{S}$$

and the Hamilton-Jacobi equation for Hamilton's principal function S: $\mathbf{R} \to \mathbf{R}$:

$$\frac{1}{2m}(S')^2 + V = E. \tag{H-J}$$

The Hamilton-Jacobi equation is related to Hamilton's equations

$$\dot{x} = \frac{\partial H}{\partial p}, \quad \dot{p} = -\frac{\partial H}{\partial x} \tag{H}$$

where $H(x, p) = p^2/2m + V(x)$, as follows. Let $\dot{x} = p/m = \partial H/\partial p$ and let