14. H. J. Sussmann, *Catastrophe theory*, Synthèse **31** (1975), 229–270.

15. H. J. Sussmann and R. S. Zahler, *Catastrophe theory as applied to the social and biological sciences: a critique*, Synthèse **37** (1978), 117–216.

16. F. Takens, *Singularities of vector fields*, Inst. Hautes Étude Sci. Publ. Math. **43** (1973), 47–100.

17. R. Thom, *Stabilité structurelle et morphogénèse*, Benjamin, Reading, Mass., 1972; English transl. (by D. Fowler), 1975.

18. J. M. T. Thompson and G. W. Hunt, *A general theory of elastic stability*, Wiley, London, 1973.

19. ______, *Towards a unified bifurcation theory*, J. Appl. Math. Phys. **26** (1975), 581–603.

20. E. C. Zeeman, *Catastrophe theory*, Sci. Amer. **234** (1976), 65–83.

21. ______, *Catastrophe theory, Selected papers*, 1972–1977, Addison-Wesley, Reading, Mass., 1977

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Innovation processes, by Yuriy A. Rozanov, John Wiley & Sons, New York, Toronto, London, and Sydney, 1977, vii + 136 pp., $14.50.

Complex valued random variables ξ_s, $s \in I$, with finite second moments being simply L_2-functions on a probability space, problems involving only the second moments are naturally set in the corresponding Hilbert space, the expectation $E[\xi_s\bar{\xi}_t] \equiv B(s, t)$ serving as the inner product. Throughout this review all random variables will be assumed to have second moments, and also, for convenience only, the first moment will be taken to be zero. The parameter set I will be taken to be the interval (α, β) on the line, where α or β may be infinite. $B(s, t)$ is the covariance function. Then $(\xi_s, s \in I)$ is a stochastic process; or a curve in Hilbert space. For $t \in I$, let $H_t(\xi)$ be the closed linear hull of $\{\xi_s: \alpha < s \leqslant t\}$, let $H(\xi)$ be the closure of the union of the $H_t(\xi)$, $t \in I$, and let P_t be the operator of orthogonal projection onto $H_t(\xi)$. The theme of Rozanov's book is the temporal evolution of the family of nondecreasing subspaces $(H_t(\xi), t \in I)$. This leads to questions in the geometry of Hilbert space naturally motivated by probabilistic considerations: $\{\xi_s: \alpha < s \leqslant t\}$ represents the observations available up to time t, and for $t < u < \beta$, $P_t\xi_u$ is the best linear predictor (in the sense of mean square error) of ξ_u in terms of the past up to time t. The process $(\xi_t, t \in I)$, will be assumed left-continuous, and this implies the same property for the family $(H_t, t \in I)$ and also the separability of $H(\xi)$. For simplicity ξ_t is taken to be complex-valued, but much recent work in the area has been devoted to vector space valued cases, and this is also the setting of Rozanov's book.

The notion of an innovations process associated with $(\xi_t, t \in I)$ is due to Cramèr [1], [2], [3]; for a related development see Hida [7]. It can be shown that there exists a finite or infinite sequence $\zeta^{(i)}$ of elements of $H(\xi)$ so that on putting $\zeta_t^{(i)} = P_t\zeta^{(i)}$ the following conditions hold: (i) $H_t(\zeta^{(i)}) \perp H_t(\zeta^{(j)})$ for $i \neq j$; (ii) setting $F_t^{(i)} = E[|\zeta_t^{(i)}|^2]$, $F^{(j)}$ is absolutely continuous with respect to $F^{(i)}$ for $j > i$; $H_t(\xi) = \sum_i \oplus H_t(\zeta_i)$. Of course each $(\zeta_t^{(i)}, t \in I)$ is a process with orthogonal increments. The length of the sequence $\zeta^{(i)}$ is called

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