

## INVARIANTS OF FINITE GROUPS AND THEIR APPLICATIONS TO COMBINATORICS

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**1. Introduction.** The theory of invariants of finite groups forms an interesting and relatively self-contained nook in the imposing edifice of commutative algebra. Moreover, there are close connections between this subject and combinatorics, for two reasons: (a) the highly combinatorial tool of *generating functions* pervades the study of invariants of finite groups, and (b) several direct applications of invariants of finite groups have recently been given to combinatorics. Here we give an exposition of the theory of invariants of finite groups with emphasis on the connections with combinatorics, which assumes a minimal background in commutative algebra and combinatorics on the part of the reader. It is hoped that such an exposition will appeal to several types of readers. (a) Those who simply wish to see a self-contained treatment of an elegant and fascinating subject. This might include coding theorists, physicists, and others who are beginning to use invariant theory as a tool in their own work. (b) Those who are interested in learning something about the revolutionary developments in present-day combinatorics. Until recently combinatorics has been regarded as a disparate collection of *ad hoc* tricks, but this picture is slowly changing under a determined effort to unify various branches of combinatorics and to understand their relationship with other branches of mathematics. (c) Finally, those who would like a relatively painless glimpse of certain topics of current interest in commutative algebra, such as the theory of Cohen-Macaulay rings and Gorenstein rings. For a really adequate understanding of these concepts it would be necessary to work in far greater generality and to introduce sophisticated machinery from

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