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HOMOLOGY STABILITY OF GL_n OF A DEDEKIND DOMAIN[1]

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The purpose of this paper is to prove that for a Dedekind domain Λ, the homomorphisms

$$H_i(GL_n(\Lambda); \mathbf{Z}) \longrightarrow H_i(GL_{n+1}(\Lambda); \mathbf{Z})$$

are isomorphisms for n sufficiently large. This problem has been of particular interest to K-theorists since $K_i(\Lambda) = \Pi_i(BGL^+(\Lambda))$ where $BGL^+(\Lambda)$ is a topological space with the property that $H_*(BGL^+(\Lambda)) \cong H_*(GL(\Lambda)) \cong \varinjlim H_*(GL_n(\Lambda))$. In particular, if Λ is a ring of algebraic integers in a number field, then the groups $GL_n(\Lambda)$ are algebraic groups, and the stability theorem allows us to apply the well-developed theory of algebraic groups toward computations of K-groups. For example, Borel-Serre [2] show that for these rings, $GL_n(\Lambda)$ has finitely generated homology. Stability then implies that $H_i(Gl(\Lambda))$ and hence $K_i(\Lambda)$ is finitely generated, thus giving a new proof of a theorem of Quillen [5]. In addition, for $\Lambda = \mathbf{Z}$, a good deal is known about p-torsion in $GL_n(\mathbf{Z})$. It is conjectured that this will give p-torsion information about $H_*(GL_n(\mathbf{Z}))$ and hence, via the results of this paper, about $K_i(\mathbf{Z})$. (At the moment, the connection between torsion in a group and torsion in the homology of the group is not entirely understood, but considerable progress in this direction has been made by K. S. Brown [4] and C. Soulé [8].)

The exact statements of the main theorems are as follows:

THEOREM 1. *For V, W finitely generated projective modules over a Dedekind domain,*

(i) $H_k(\mathrm{Aut}(W \oplus V), \mathrm{Aut}(W); \mathbf{Z}) = 0$ *for* $\mathrm{rk}\ W \geqslant 4k + 1$,

(ii) $H_k(\mathrm{Aut}(W \oplus V), \mathrm{Aut}(W); \mathbf{Z}[\tfrac{1}{2}]) = 0$ *for* $\mathrm{rk}\ W \geqslant 3k + 1$.

THEOREM 2. *For Λ a PID, $G_n = GL_n(\Lambda)$ or $SL_n(\Lambda)$*

(i) $H_k(G_{n+1}(\Lambda), G_n(\Lambda); \mathbf{Z}) = 0$ *for* $n \geqslant 3k$,

(ii) $H_k(G_{n+1}(\Lambda), G_n(\Lambda); \mathbf{Z}[\tfrac{1}{2}]) = 0$ *for* $n \geqslant 2k$.

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