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ERGODIC THEOREMS IN DEMOGRAPHY

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ABSTRACT. The ergodic theorems of demography describe the properties of a product of certain nonnegative matrices, in the limit as the number of matrix factors in the product becomes large. This paper reviews these theorems and, where possible, their empirical usefulness. The strong ergodic theorem of demography assumes fixed age-specific birth and death rates. An approach to a stable age structure and to an exponentially changing total population size, predicted by the Perron-Frobenius theorem, is observed in at least some human populations. The weak ergodic theorem of demography assumes a deterministic sequence of changing birth and death rates, and predicts that two populations with initially different age structures will have age structures which differ by less and less. Strong and weak stochastic ergodic theorems assume that the birth and death rates are chosen by time-homogeneous or time-inhomogeneous Markov chains and describe the probability distribution of age structure and measures of the growth of total population size. These stochastic models and theorems suggest a scheme for incorporating historical human data into a new method of population projection. The empirical merit of this scheme in competition with existing methods of projection remains to be determined. Most analytical results developed for products of random matrices in demography apply to a variety of other fields where products of random matrices are a useful model.

1. Introduction. According to his autobiography, Ulam [**1976**, p. 6] once introduced himself as a pure mathematician who had sunk so low that his latest paper contained numbers with decimal points. This paper will sink–if possible–even lower, to pictures of numbers with decimal points. The reasons are that I make no pretense of being a pure mathematician (although some of my best friends are) and that I will describe a young, not a mature, field of science. This field is still very close to its empirical roots. Consequently, even the mathematical parts of this paper will be framed in concrete language. Many of the assumptions made here can be weakened, at the cost of more technicalities.

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