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SELFADJOINT OPERATOR EXTENSIONS SATISFYING THE WEYL COMMUTATION RELATIONS

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ABSTRACT. Motivated by questions concerning uniqueness of unbounded derivations in commutative C^*-algebras, and related problems on singular perturbations, we define two mixed global and infinitesimal versions of the Weyl operator commutation relations (one degree of freedom and infinite multiplicity), a weak one and a strong one. We announce two structure theorems of a geometric nature which characterize the nonselfadjoint symmetric operators entering in the Weyl systems. Proofs are only indicated.

Our starting point is the following variant of the Stone-von Neumann Uniqueness Theorem [12], [4b]. Let (U, V) be a pair of unitary one-parameter groups (always assumed strongly continuous) of operators on a separable Hilbert space $\mathcal{H}$, and suppose that the Weyl commutation relation

$$U(t)V(s) = V(s)U(t)e^{its} \quad \text{(for all } s, t \in \mathbf{R}) \tag{1}$$

holds. Then it is possible to represent the system in the form $SU(t)S^{-1}f(x) = f(x + t)$, $SV(s)S^{-1}f(x) = e^{isx}f(x)$, where S is an isometry of a space $L^2(\mathbf{R}, \mathcal{N})$ of the norm-square integrable functions f, with values in a separable Hilbert space $\mathcal{N}$, onto $\mathcal{H}$; the dimension of $\mathcal{N}$ being equal to the (uniform) multiplicity of the spectrum of U.[2]

Instead of (1) we consider the following *infinitesimal* Weyl relation with symmetric but generally nonselfadjoint generator. Let $\{U(t)\}_{t\in\mathbf{R}}$ be a unitary one-parameter group on $\mathcal{H}$, and let Q be a symmetric operator with dense domain $\mathcal{D}(Q)$ in $\mathcal{H}$. The corresponding relation

$$(U(t)Qf, g) = (U(t)f, Qg) + t(U(t)f, g) \quad \text{for all } f, g \in \mathcal{D}(Q) \tag{2}$$

is here called the infinitesimal Weyl relation for the triple $(U, Q, \mathcal{H})$. It is clearly equivalent to (1) if Q is essentially selfadjoint. But in scattering theory of singular perturbations, and in recent investigations of the author concerning uniqueness of unbounded derivations, the relation (2) for nonselfadjoint Q plays an interesting role. Simple examples show that the operator Q of a given system $(U, Q, \mathcal{H})$ may

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[2] We refer to [LP] as a general reference, containing in addition important applications.

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