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HOMOTOPY INVERSES FOR NERVE

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Whitehead [19] introduced the category of CW complexes as the appropriate category in which to do homotopy theory. Eilenberg, Mac Lane and Zilber ([1], [2]) defined the notion of simplicial set in the early 50's and Kan ([5], [6], [7]) introduced the necessary conditions to do homotopy theory in this category. The equivalence of these categories under adjoint functors (see [14], [6], [5], [4]) played an important role in the development of geometric topology. In the late 60's, Quillen [16] used the notion of classifying space for a small category [17], and showed the importance of doing homotopy theory in the category of small categories. Latch [8] recently showed that the category of small categories and the category of simplicial sets were equivalent "up to homotopy," but not by using adjoint functors. In this paper, adjoint pairs are given and a general criteria for such adjoint functors to induce a "homotopy equivalence" are announced.

The homotopic category of $\mathcal{K}$, the category of (semi-) simplicial sets, is equivalent to the homotopy category of $\mathcal{W}$, the category of spaces of homotopy type of a CW complex [4, VII, 1], via a pair of adjoint functors. Moreover, in [8], the homotopic categories of $\mathcal{K}$ and $\mathcal{C}$at, the category of small categories, are shown to be equivalent via the pair N: $\mathcal{C}\mathrm{at} \longrightarrow \mathcal{K}$ and Γ: $\mathcal{K} \longrightarrow \mathcal{C}\mathrm{at}$, where N is the standard embedding nerve functor and Γ is the category of simplices functor. As in the case for $\mathcal{K}$ and $\mathcal{W}$, one would like to replace Γ by the left adjoint of nerve, categorical realization c: $\mathcal{K} \longrightarrow \mathcal{C}\mathrm{at}$; however, c is "wildly wrong" with respect to homotopy since it maps certain spheres to contractible categories. In this announcement, we give conditions for other "reasonable" functors from $\mathcal{K}$ to $\mathcal{C}$at to be (weak) homotopy inverses for nerve. The functor Γ: $\mathcal{K} \longrightarrow \mathcal{C}\mathrm{at}$ above and the functor Λ: $\mathcal{K} \longrightarrow \mathcal{C}\mathrm{at}$ used in [11] are examples of such homotopy inverses.

We only consider functors from $\mathcal{K}$ to $\mathcal{C}$at having right adjoints. Under very weak homotopy conditions, these right adjoints are homotopy equivalent to nerve

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