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RESEARCH ANNOUNCEMENTS

CLASSIFICATION OF THE IRREDUCIBLE REPRESENTATIONS OF $\mathfrak{sl}(2, \mathbf{C})$

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Let $\mathfrak{g}$ be a nonabelian Lie algebra over an algebraically closed field K of characteristic 0. One is interested in the (algebraically) irreducible representations of $\mathfrak{g}$ acting on a vector space which is allowed to be infinite dimensional. The subject of enveloping algebras is largely concerned with these, but even in the simplest nonabelian case, with $\mathfrak{g} = \mathfrak{h}$ the 3-dimensional (nilpotent) Heisenberg algebra, as Dixmier remarks in discussing the situation when $K = \mathbf{C}$ in the preface to [2], "a deeper study reveals the existence of an enormous number of irreducible representations of $\mathfrak{h}$ . . . It seems that these representations defy classification. A similar phenomenon exists for $\mathfrak{g} = \mathfrak{sl}(2)$, and most certainly for all noncommutative Lie algebras."

However, as we shall see, the situation for $\mathfrak{h}$ and for $\mathfrak{sl}(2)$ turns out to be far nicer than hoped for. Indeed we announce here a determination and classification of all irreducible representations of $\mathfrak{h}$, of $\mathfrak{sl}(2)$, and of the 2-dimensional nonabelian Lie algebra, and thus of the prototypes respectively of nilpotent, simple, and solvable Lie algebras. As a guide to the meaning of "classification" and because our results use the same invariants, consider a classical situation of an (associative) algebra for which the irreducible representations have long been classified, namely, the algebra B of formal linear differential operators with rational function coefficients, i.e., $B = K(q)[p]$, the (noncommutative) polynomials in an indeterminate p where multiplication is determined by the relation $pq - qp = 1$. Then B is a left principal ideal domain. Therefore [3] a B-module M is simple if and only if $M \cong B/Bb$ for some $b \in B$ which is irreducible (i.e., $b = ac$ implies a or c is a unit); and $B/Bb \cong B/Ba$ if and only if a and b are similar, i.e., there exists $c \in B$ such that $(b, c) = 1$ and $a = [b, c]c^{-1}$ where (b, c) is a

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