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Proof theory, by Kurt Schütte, Grundlehren der Mathematischen Wissenschaften, Band 225, Springer-Verlag, Berlin, Heidelberg, New York, 1977, xii + 302 pp., $34.10.

This is the long-awaited translated revision of the author's *Beweistheorie* [8]. As Schütte says in his preface, it was originally intended to be the second edition of that book "but in fact has been completely rewritten". Even so the amount of fresh material is quite impressive. The obvious comparisons to be made are with the first edition of [8] and with Takeuti's *Proof theory* [12] previously reviewed in this journal [4]. These are the main candidates for contemporary texts on the subject which essay some comprehensiveness. Very briefly, there is much to admire and to welcome in this new book, both with respect to choice of material and to manner of presentation. As to the latter, we have (as expected from the author) meticulous attention to details, careful definitions, complete but compact proofs, and assimilable organization into units and subunits. However, I cannot recommend the book unreservedly, for reasons to be gone into below.

There are three parts: A. Pure Logic, B. Systems of Arithmetic, and C. Subsystems of Analysis. The subject matter of A is by now fairly standard, but the formal framework in terms of *positive* and *negative parts* is not, even though it had been introduced in [8]. The idea is that A is a positive (negative) part of F if it occurs in F in such a way that the truth of A (falsity of A) implies the truth of F. The simplest example is given by $F = (A_1 \wedge \cdots \wedge A_n \to B_1 \vee \cdots \vee B_m)$ where each A_i is a negative part of F and each B_j a positive part; similarly for $F = (\neg A_1 \vee \cdots \vee \neg A_n \vee \cdots \vee B_m)$. The syntactic definition of these notions is given in terms of *P-forms* $\mathcal{P}[*]$ and *N-forms* $\mathcal{N}[*]$ so that A is a positive part of F if $F = \mathcal{P}[A]$ for a P-form $\mathcal{P}$ (similarly for negative parts and N-forms). It is also necessary for setting up the logical systems to consider *NP-forms* $\mathcal{Q}[*_1, *_2]$ which are N-forms in $*_1$ and P-forms in $*_2$. These notions are supposed to permit combining the advantages of Hilbert-style inferential systems with those of Gentzen-style sequential systems. In the former, as here, one infers individual formulas while in the latter one infers "sequents" $\Gamma \supset \Delta$ where $\Gamma = \{A_1, \ldots, A_n\}$, $\Delta = \{B_1, \ldots, B_m\}$ are sequences or sets of formulas; $\Gamma \supset \Delta$ holds under the same conditions as $A_1 \wedge \cdots \wedge A_n \to B_1 \vee \cdots \vee B_m$.

It is of interest to compare the two systems of axioms and rules for the classical predicate calculus CP, taken here with basic logical symbols $\perp$, $\to$, $\forall$. In Schütte's formulation these are: $(Ax.\mathrm{I})\mathcal{Q}[A, A]$, $(Ax.\mathrm{II})\mathcal{N}[\perp]$, $(\to \mathrm{R1})\mathcal{N}[A \to \perp]$, $\mathcal{N}[B] \vdash \mathcal{N}[A \to B]$, $(\forall \mathrm{R1})\mathcal{P}[F(a)] \vdash \mathcal{P}[\forall x F(x)]$ (when a is not free in the conclusion), and $(\forall \mathrm{R2})\forall x F(x) \to \mathcal{N}[F(t)] \vdash \mathcal{N}[\forall x F(x)]$. In Gentzen's formulation these are: $(Ax.\mathrm{I})'(\Gamma, A \supset \Delta, A)$, $(Ax.\mathrm{II})'(\Gamma, \perp \supset \Delta)$, $(\to \mathrm{R1})'(\Gamma \supset \Delta; A)$, $(\Gamma, B \supset \Delta) \vdash (\Gamma, A \to B \supset \Delta)$, $(\to \mathrm{R2})'(\Gamma, A \supset \Delta, B) \vdash (\Gamma \supset \Delta, A \to B)$, $(\forall \mathrm{R1})'(\Gamma \supset \Delta, F(a)) \vdash (\Gamma \supset \Delta, \forall x F(x))$, (when a is not free in the conclusion), and $(\forall \mathrm{R2})'(\Gamma, F(t) \supset \Delta) \vdash (\Gamma, \forall x F(x) \supset A)$. In