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Nonlinearity and functional analysis, Melvyn S. Berger, Academic Press, New York, San Francisco, London, 1977, xix + 415 pp., $24.50.

Linear functional analysis evolved as the natural gathering point for a number of different investigations into the solvability of linear equations which were either in the form of integral equations or in the form of countable systems of linear scalar equations in which the unknown was a sequence of numbers. As the subject developed much broader areas of applicability became evident. These applications, in turn, spawned further abstract development, and the abstract results themselves assumed an intrinsic interest. The basic approach of functional analysis, where one considers functions to be points in a large space of related functions and lets the differential or integral operator act on these points, has also been very successful in treating nonlinear problems.

The division between the linear and the nonlinear theory is, of course, not so sharp. As we know from calculus, a great deal of information about a system of nonlinear equations is obtained from their local linear approximation. Moreover, it is often possible to glean information about a linear problem by considering a related nonlinear problem; this is so strikingly demonstrated in Lomonosov's recent results on the invariant subspace problem for linear operators [5].

Berger's aim is to give a systematic treatment of some of the fundamental abstract nonlinear results and of their application to certain concrete problems in geometry and physics.

The study of nonlinear operators acting on infinite dimensional spaces has an obvious starting point–study the finite-dimensional case, a finite system of scalar equations in a finite number of unknowns. Even at this step we note that a fairly complete description of the solutions would be very difficult; when $p(x)$ is a polynomial in a single variable the study of solutions of $p(x) = 0$ is the subject matter of classical algebraic geometry.

Let n and k be integers, $n \geqslant 1$, $k + n \geqslant 1$ and let $\mathcal{O}$ be a bounded open subset of $\mathbf{R}^n$. Suppose $f\colon \mathcal{O} \to \mathbf{R}^{n+k}$ is continuous.

If f is one-to-one then $k \geqslant 0$. Moreover, assuming f is one-to-one, $f(\mathcal{O})$ is open in $\mathbf{R}^{n+k}$ iff $k = 0$. (F1)