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AN INTRODUCTION TO THE CLASSIFICATION OF FINITE SIMPLE GROUPS

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The classification of the finite simple groups now seems close at hand. You may judge for yourself just how close from Daniel Gorenstein's article in this Bulletin, introduced below.

One of the most important techniques in modern science and mathematics is the investigation of an object from the point of view of its symmetry group. This approach originated in Galois' investigation of number fields, which lead to a proof of the insolvability of the quintic. The groups considered by Galois were finite, as are the groups arising in many other interesting problems. This is fortunate as finite groups are much more tractable than infinite groups. For example it is difficult to imagine that a classification of the infinite simple groups will ever be achieved.

The axioms describing finite groups are concise and natural. They lead to a truly rich structure represented by a large and satisfying family of examples. The finite simple groups are particularly interesting. The groups of Lie type are the finite analogues of the semisimple algebraic and Lie groups. Following Tits, the groups of Lie type may be defined as the finite groups G with a (B, N)-pair, where B is the normalizer of some Sylow group of G which is a complement to $B \cap N$ in B. Consider for example the group $GL(n, F)$ of n by n nonsingular matrices over F. If F is the complex numbers, $GL(n, F)$ is a Lie group. If F is a finite field, $GL(n, F)$ is a group of Lie type. The groups of Lie type come equipped with an important geometric structure, the Tits building. The building of $GL(n, F)$ is the simplicial complex whose points are the collection of subspaces of an n-dimensional vector space over F, with simplices defined by inclusion. While each semisimple Lie group has its analogue among the finite simple groups, the converse is not true. In addition to the ordinary groups of Lie type there are certain twisted groups of Lie type with no continuous analogue, there are the groups of prime order, the alternating groups, and finally there are twenty-four further finite simple groups, together with overwhelming evidence for the existence of yet two more. The last twenty-six groups are called the sporadic groups. Many mathematicians other than simple group theorists find them fascinating.

Of course the simple groups are the building blocks of the finite groups. Determining the simple groups and their internal properties leave many questions about finite groups unanswered but makes possible the solution of many others. I mention only three examples, each over half a century old. The first is a conjecture of Schreier that the outer automorphism group of a

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