difficult." Indeed, we do. After studying this chapter, mathematicians will find these discussions less difficult.

Prerequisites for profitable reading of this book consist of a knowledge of basic graduate level differential geometry and a knowledge of Newtonian physics at least equivalent to a good freshman level course in the subject. The presence of many exercises make the book appropriate as a text, perhaps for example as the second semester of a first graduate course in differential geometry. The first half of the book should already sufficiently prepare the reader for entry into the physics literature with a minimum of trauma.

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Stochastic integration and generalized martingales, by A. U. Kussmaul, Pitman Publishing, London, San Francisco, Melbourne, 1977, 163 pp., $14.00.

Modern stochastic integration began in 1828 when the English botanist Robert Brown observed the motion of pollen grains in a glass of water. Bachelier (1900, [2]) and Einstein (1905, [7]) studied the mathematical modelling of the motion of the grains, now called *Brownian motion*. But it was Wiener (1923, [25]), making use of the ideas of Borel and Lebesgue, who created the modern rigorous mathematical model of Brownian motion, the *Wiener process*.

The Wiener process has three properties which make it of fundamental importance to the theory of stochastic processes: it is a Gaussian process, it is a strong Markov process, and it is a martingale. Let $W = (W(t, \omega))_{t \geq 0}$ denote a Wiener process, in which t is the time and each ω is a particle; then $W(t, \omega)$ represents the position of that particle at time t. One can show that except on a set of probability zero, every sample path (i.e., $W(t, \omega)$ as a function of t for fixed ω) is continuous but is of unbounded variation on every compact time set. Since the sample paths are nowhere differentiable, the Brownian particle cannot have an instantaneous velocity. This bizarre property may be viewed as a consequence of the Markov property; that is: