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PATTERN FORMATION AND PERIODIC STRUCTURES IN SYSTEMS MODELED BY REACTION-DIFFUSION EQUATIONS

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1. Introduction. Excitable media are most often modeled by "reaction-diffusion" equations of the form

$$\partial \mathbf{u}/\partial t = \mathbf{F}(\mathbf{u}) + \Lambda\Delta\mathbf{u}. \tag{1.1}$$

$\mathbf{u}(\mathbf{x}, t)$ is an n vector which defines the state of the system at a given point $\mathbf{x}$ at time t. The nonlinear function $\mathbf{F}$ describes the "kinetics" of the medium while the diffusion effects enter via the term $\Lambda\Delta\mathbf{u}$. Λ is a diagonal matrix with nonnegative entries and Δ is the Laplace operator in the appropriate number of space dimensions, in this paper one or two. In a spatially homogeneous configuration the state of the system evolves according to

$$d\mathbf{u}/dt = \mathbf{F}(\mathbf{u}). \tag{1.2}$$

The features which characterize "excitability" can be described in terms of this kinetic equation. They are (i) a globally stable equilibrium or rest state, (ii) an "excited" region of the state space which can be reached if a stimulus is applied to the system which exceeds some threshold level, and (iii) a refractory region of the state space in which the system will gradually return to rest, and will not respond to further stimulation unless sufficiently near the rest state. The behavior of a typical trajectory is summarized in Figure 1.1.

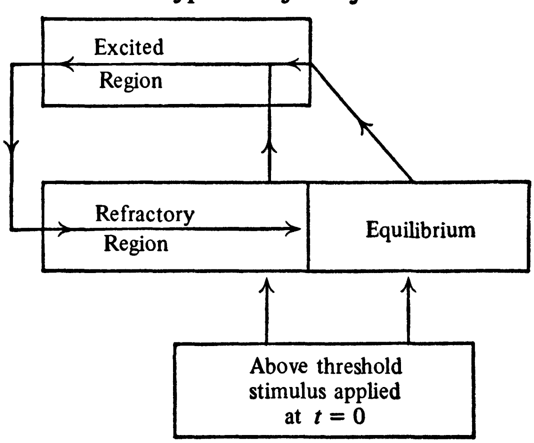

FIGURE 1.1

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