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THE ISOPERIMETRIC INEQUALITY

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The circle is uniquely characterized by the property that among all simple closed plane curves of given length L, the circle of circumference L encloses maximum area. This property is most succintly expressed in the isoperimetric inequality

$$L^2 \geqslant 4\pi A, \tag{1}$$

where A is the area enclosed by a curve C of length L, and where equality holds if and only if C is a circle.

The purpose of this paper is to recount some of the most interesting of the many sharpened forms, generalizations, and applications of this inequality, with emphasis on recent contributions. Earlier work is summarized in the book of Hadwiger [**1**]. Other general references, varying from very elementary to quite technical are Kazarinoff [**1**], Pólya [**2**, Chapter X], Porter [**1**], and the books of Blaschke listed in the bibliography. Most books on convexity also contain a discussion of the isoperimetric inequality from that perspective. One aspect of the subject is given by Burago [**1**]. Others may be found in a recent paper of the author [**4**] on Bonnesen inequalities and in the book of Santaló [**4**] on integral geometry and geometric probability.

An important note: we shall *not* go into the area of so-called "isoperimetric problems". Those are simply variational problems with constraints, whose name derives from the fact that inequality (1) corresponds to the first example of such a problem: maximize the area of a domain under the constraint that the length of its boundary be fixed. There are also the "isoperimetric inequalities" of mathematical physics. They are special cases of isoperimetric problems in which typically some physical quantity, usually represented by the eigenvalues of a differential equation, is shown to be extremal for a circular or spherical domain. Extensive discussions of such problems can be found in the book of Pólya and Szegö [**1**] and the review article by Payne [**1**]. We shall discuss them here only insofar as they relate to the main subject of this paper.

What we shall concentrate on here is "the" isoperimetric inequality (1) and other geometric versions and generalizations of it. We shall also consider

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