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QUALITATIVE METHODS IN BIFURCATION THEORY

Bifurcation decreases entropy
. . . Helen Petard

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Classical bifurcation theory is undergoing a revitalization with the infusion of ideas from singularities of mappings and structural stability. The situation now is similar to that a quarter century ago when Krasnosel'skiĭ introduced topological methods, especially degree theory, into the subject (see Krasnosel'skiĭ [1964]). Like degree theory, the theory of singularities of mappings is playing a fundamental role in the development of the subject.

Our goal is to give a few examples of how qualitative ideas can give insight into bifurcation problems. The literature and full scope of the theory is too vast to even attempt to survey here. On the classical bifurcation theory side, the survey article of Sather [1973] is valuable, and for the theory of singularities of mappings, we refer to Golubitsky and Guillemin [1973]. On the overlap, the article of Hale [1977] is recommended.

The credit for using ideas of singularities of mappings and structural stability in bifurcation theory is often attributed to Thom (see Thom [1972]) and on the engineering side, to Thompson and Hunt [1973], Roorda [1965] and Sewell [1966]. However, to penetrate the classical bifurcation circuit is another matter. For this, there are a number of recent articles, notably, Chillingworth [1975], Chow, Hale and Mallet-Paret [1975], Magnus and Poston [1977], Holmes [1977] and Potier-Ferry [1977]. The literature on this interaction is in an explosive state and we merely refer to the above articles, Chillingworth [1976], Golubitsky [1978], Marsden and McCracken [1976], Abraham and Marsden [1978] and Poston and Stewart [1978] for further references.

1. The definition of bifurcation point. The very definition of bifurcation point varies from author to author, although in any specific situation there is usually no doubt about what should be called a bifurcation point.

The "classical" definition is typified by the following discussion in Matkowski and Reiss [1977]:

> "Bifurcation theory is a study of the branching of solutions of nonlinear equations $f(x, \lambda) = 0$ where f is a nonlinear operator, x is the solution vector and λ is a

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